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New Aspects in Human Middle Ear Dynamics: Modelling and Analysis of Intact and Implanted Ossicular Chain

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Doctoral thesis

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Chciałbym złożyć serdeczne podziękowania mojej żonie Weronice za nieustanne wsparcie, cierpliwość i wiarę we mnie przez cały okres pisania doktoratu. Szczególne podziękowania kieruję do mojego promotora, Pana prof. dr hab. inż. Rafała Rusinka, za cenne wskazówki, merytoryczne uwagi oraz inspirujące rozmowy, które znacząco wpłynęły na kształt niniejszej pracy. Dziękuję również koleżankom i kolegom z Katedry Mechaniki Stosowanej za życzliwą atmosferę, pomoc oraz motywujące dyskusje, które towarzyszyły mi na różnych etapach badań.

Table 1. *List of acronyms and abbreviation.*

Abbreviation	Meaning
AL	Annular Ligament
AML	Anterior Malleolar Ligament
DOF	Degree of Freedom
FMT	Floating Mass Transducer
FRF	Frequency Response Function
HME	Human Middle Ear
H_{TD}	Transfer function of the stapes displacement to sound pressure
H_{TV}	Transfer function of the stapes velocity to sound pressure
IME	Implanted Middle Ear
IMEHD	Implantable Middle Ear Hearing Device
IMJ	Incudomalleolar Joint
IOC	Intact Ossicular Chain
ISJ	Incudostapedial Joint
LDV	Laser Doppler Vibrometry
LPM	Lumped Parameter Model
ME	Middle Ear
PIL	Posterior Incudal Ligament
SF	Stapes Footplate
TM	Tympanic Membrane

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Abstract/Streszczenie

ABSTRACT

Hearing plays an indispensable role in human communication and environmental awareness, necessitating continuous research into auditory biomechanics to better understand sound transmission processes. This dissertation investigates novel aspects of human middle ear modeling, focusing on the mechanics of ossicle chain e.g. with relaxation effect. By examining both intact and implanted middle ear conditions through experimental research and computational modeling, this study advances the current understanding of middle ear mechanics, particularly in the context of ligament relaxation, nonlinear mechanical behaviors, and double excitation phenomena in implanted ears. The research problem centers on the vibrational dynamics of the middle ear, particularly in relation to the relaxation effects of ligaments and tendons, the behavior of a Floating Mass Transducer (FMT) within an implanted system, and the impact of double excitation on the ossicular chain. The fundamental scientific questions addressed include: (1) How does ligament relaxation affect sound transfer through the middle ear? (2) What role do nonlinear mechanical properties of an FMT play in altering the vibrational response of the implanted middle ear? (3) How does double excitation, coupled with the delay in implant signal processing, influence sound transmission? To explore these issues, the study employs a combination of experimental and numerical approaches, integrating lumped parameter models with computational analysis of sound wave propaga-

tion. A comprehensive literature review underscores the significance of modeling the auditory system, delineating the anatomy and mechanics of the middle ear. The human ear comprises three main parts: the outer ear, the middle ear, and the inner ear. The middle ear is of particular interest due to its role in sound transmission via three tiny bones—malleus, incus, and stapes—which form a mechanical bridge between the eardrum and cochlea. The study explores existing models and identifies gaps, particularly in the representation of viscoelastic ligament relaxation, nonlinear stiffness effects, and bistable behavior in middle ear implants. To address these limitations, the research develops lumped parameter models (LPMs) of the middle ear, employing both Kelvin-Voigt and Maxwell viscoelastic models to simulate ligament relaxation effects. The study presents a three-degree-of-freedom (3-DOF) model for an intact middle ear and two five-degree-of-freedom (5-DOF) models for an implanted middle ear, one incorporating single excitation and the other incorporating double excitation with time delay effects. The Maxwell viscoelastic model introduces a modified representation of tissue elasticity, incorporating relaxation effects into the mechanical response of ligaments and tendons. The developed models demonstrate that relaxation phenomena significantly influence stapes vibration and, consequently, auditory signal transmission. Experimental research plays a crucial role in validating the numerical models. The study employs human temporal bones to examine middle ear vibrations under different excitation conditions. Bistability emerges in implanted middle ear models, indicating that multiple stable states of vibration can exist within the system. This finding has significant implications for the design and optimization of middle ear implants, as bistability can lead to variations in auditory perception. The dissertation further explores the effects of double excitation, wherein the implanted middle ear receives stimulation both from the eardrum and from the mechanical input of the FMT. Numerical simulations indicate that double excitation leads to complex vibrational interactions, with significant implications for implant performance. Additionally, time delay in the implant's electrical processing introduces phase shifts that alter stapes footplate vibration. These results highlight the importance of accounting for double excitation and time delay effects when designing middle ear prostheses. Key findings from the study include: 1. Relaxation Effects: The viscoelastic properties of ligaments and tendons significantly influence sound transmission, with

time-dependent elasticity affecting ossicular chain dynamics. 2. Bistability in Implants: The nonlinear mechanical behavior of FMTs introduces bistable states, which must be carefully managed to optimize implant performance. 3. Double Excitation and Time Delay: The interaction between natural sound conduction and implant-generated mechanical input creates complex vibrational patterns, requiring careful calibration of implant signal processing. The study makes several contributions to the field of auditory biomechanics. First, it extends existing middle ear models by incorporating ligament relaxation effects, providing a more accurate representation of physiological behavior. Second, it identifies bistability as a critical factor in middle ear implants, emphasizing the need for further research into nonlinear vibrational dynamics. Third, it highlights the significance of double excitation and time delay effects, offering practical insights for improving implant design and auditory prostheses. In conclusion, this dissertation advances the understanding of human middle ear mechanics through a combination of experimental and computational approaches. By integrating viscoelastic models, nonlinear dynamics, and implant signal processing considerations, the study provides a comprehensive framework for analyzing middle ear function in both natural and surgically altered states. These findings have far-reaching implications for the diagnosis, treatment, and rehabilitation of hearing disorders, contributing to the development of more effective middle ear implants and auditory prostheses. Future research should further refine these models by incorporating three-dimensional finite element simulations and expanding experimental studies to include a broader range of middle ear conditions. Through continued innovation in middle ear modeling, the field of auditory biomechanics can move closer to developing optimal solutions for individuals with hearing impairments. Additionally, expanding the research into real-world clinical applications can pave the way for improved auditory prosthetic devices, including next-generation cochlear implants and middle ear implants that account for personalized anatomical variations. The integration of machine learning and artificial intelligence in computational modeling may further enhance predictive accuracy, enabling medical professionals to tailor interventions with unprecedented precision. The significance of this research extends beyond the laboratory, offering potential breakthroughs in audiology, otolaryngology, and biomedical engineering. By fostering interdisciplinary collaboration, this study serves

as a foundation for future advancements in auditory science, ultimately improving the quality of life for millions of individuals affected by hearing impairments worldwide. Further exploration of bio-compatible materials and real-time adaptive prosthetics could revolutionize hearing aid technology, leading to more responsive and natural auditory experiences for users. These advancements will be pivotal in bridging the gap between theoretical research and practical implementation, ensuring that cutting-edge developments reach those in need effectively and efficiently.

STRESZCZENIE

Słuch odgrywa niezastąpioną rolę w komunikacji międzyludzkiej oraz w percepcji środowiska, co wymaga ciągłych badań nad biomechaniką słuchu w celu lepszego zrozumienia procesów transmisji dźwięku. Niniejsza rozprawa analizuje nowe aspekty w symulacjach modeli ludzkiego ucha środkowego, koncentrując się na mechanice drgań łańcucha kosteczek słuchowych. Badanie obejmuje zarówno zdrowe ucho, jak i ucho środkowe z implantem, wykorzystując badania eksperymentalne oraz obliczenia numeryczne, aby poszerzyć wiedzę na temat mechaniki ucha środkowego, szczególnie w kontekście relaksacji więzadeł, nieliniowych właściwości mechanicznych oraz zjawiska podwójnego wymuszenia w uchu z implantem.

Problem badawczy koncentruje się na dynamice drgań ucha środkowego, zwłaszcza w odniesieniu do efektów relaksacji więzadeł i ścięgien, zachowania wzbudnika będącego masą płwającą (FMT) w uchu z implantem oraz wpływu podwójnego wymuszenia na łańcuch kosteczek słuchowych. Kluczowe pytania naukowe obejmują: (1) W jaki sposób relaksacja więzadeł wpływa na przenoszenie dźwięku przez ucho środkowe? (2) Jakie znaczenie mają nieliniowe właściwości mechaniczne FMT w modyfikacji odpowiedzi rezonansowej środkowego ucha z implantem? (3) Jak podwójne wymuszenie, w połączeniu z opóźnieniem w przetwarzaniu sygnału implantu, wpływa na transmisję dźwięku?

W celu zbadania tych zagadnień, w pracy zastosowano kompleksowe podejście łączące eksperymenty, badania numeryczne i obliczenia analityczne prowadzone na modelach dyskretnych.

Dogłębny przegląd literatury podkreśla znaczenie wykonywania modeli układu słuchowego, omawiając anatomię i mechanikę ucha środkowego. Ludzkie ucho

składa się z trzech głównych części: ucha zewnętrznego, środkowego i wewnętrznego. Szczególną uwagę zwrócono na ucho środkowe ze względu na jego kluczową rolę w transmisji dźwięku poprzez trzy drobne kosteczki—młoteczek, kowadełko i strzemiączko - tworzące mechaniczne połączenie między błoną bębenkową a ślimakiem. Praca analizuje istniejące modele i wskazuje na ich ograniczenia, szczególnie w zakresie uwzględnienia relaksacji więzadeł o właściwościach lepkosprężystych, efektów nieliniowej sztywności oraz bistabilnego zachowania implantów ucha środkowego.

Aby rozwiązać te ograniczenia, opracowano modele skupionych parametrów (LPM) ucha środkowego, wykorzystując modele lepkosprężyste Kelvina-Voigta i Maxwella do symulacji efektów relaksacji więzadeł. W pracy zaprezentowano model o trzech stopniach swobody (3-DOF) dla zdrowego ucha środkowego oraz dwa modele o pięciu stopniach swobody (5-DOF) dla ucha środkowego z implantem — jeden uwzględniający pojedyncze wzbudzenie, a drugi podwójne z efektami opóźnienia czasowego. Model lepkosprężysty Maxwella wprowadza zmodyfikowaną reprezentację elastyczności tkanek, uwzględniając efekty relaksacji w odpowiedzi mechanicznej więzadeł i ścięgien. Opracowane modele wykazują, że zjawiska relaksacyjne znacząco wpływają na drgania strzemiączka, a tym samym na transmisję sygnału słuchowego.

Badania eksperymentalne odgrywają kluczową rolę w weryfikacji modeli numerycznych. Praca wykorzystuje ludzkie kości skroniowe do analizy drgań ucha środkowego w różnych warunkach wymuszenia. W modelach ucha środkowego z implantem ujawnia się bistabilność, wskazująca na możliwość istnienia wielu stabilnych stanów drgań w systemie. To odkrycie ma istotne implikacje dla projektowania i optymalizacji implantów ucha środkowego, ponieważ bistabilność może prowadzić do różnic w percepcji słuchowej.

Rozprawa analizuje także wpływ podwójnego wymuszenia, w której ucho środkowe z implantem otrzymuje sygnały zarówno z błony bębenkowej, jak i z mechanicznego wzbudzenia przez FMT. Symulacje numeryczne wskazują, że podwójne wymuszenie prowadzi do złożonych interakcji w drganiach, mających znaczący wpływ na wydajność implantu. Dodatkowo, opóźnienie czasowe w przetwarzaniu sygnału elektrycznego przez implant wprowadza przesunięcia fazowe, które modyfikują drgania podstawy strzemiączka. Wyniki te podkreślają konieczność

uwzględnienia podwójnego wymuszenia i efektów opóźnienia czasowego podczas projektowania protez ucha środkowego.

Kluczowe wnioski badawcze obejmują:

Efekty relaksacji – Właściwości lepkosprężyste więzadeł i ścięgien znacząco wpływają na transmisję dźwięku, przy czym elastyczność zależna od czasu modyfikuje dynamikę kosteczek słuchowych. Bistabilność implantów – Nieliniowe właściwości mechaniczne FMT prowadzą do powstawania bistabilnych stanów, które muszą być kontrolowane w celu optymalizacji wydajności implantu. Podwójne wymuszenie i opóźnienie czasowe – Interakcja pomiędzy naturalnym przewodzeniem dźwięku a mechanicznym pobudzeniem implantu prowadzi do skomplikowanych wzorców drgań, wymagających precyzyjnej kalibracji przetwarzania sygnału implantu. Praca wnosi istotny wkład w dziedzinę biomechaniki słuchu. Po pierwsze, rozszerza istniejące modele ucha środkowego, uwzględniając efekty relaksacji więzadeł, co pozwala na dokładniejszą reprezentację procesów fizjologicznych. Po drugie, identyfikuje bistabilność jako kluczowy czynnik w implantach ucha środkowego, podkreślając potrzebę dalszych badań nad nieliniową dynamiką drgań. Po trzecie, wskazuje na znaczenie podwójnego wymuszenia i efektów opóźnienia czasowego, dostarczając praktycznych wskazówek dotyczących ulepszeń w projektowaniu protez słuchowych.

Podsumowując, niniejsza rozprawa przyczynia się do lepszego zrozumienia mechaniki ludzkiego ucha środkowego poprzez połączenie badań eksperymentalnych, analitycznych i numerycznych. Znaczenie tej pracy wykracza poza problematykę inżynierii mechanicznej, oferując potencjalne przełomy w audiologii, otolaryngologii i inżynierii biomedycznej. Dzięki dalszym innowacjom w modelowaniu ucha środkowego, dziedzina biomechaniki słuchu może zbliżyć się do opracowania optymalnych rozwiązań dla osób z ubytkami słuchu.

Introduction

Hearing is one of the most vital human senses, playing a crucial role in communication, environmental awareness, and survival. Over tens of thousands of years, the ability to hear has enabled humans to develop speech, perceive auditory cues, and respond to potential threats in their surroundings. In modern times, however, people are constantly exposed to auditory stimuli, both at work and at home. Urban environments, in particular, are saturated with noise from traffic and other sources. According to the World Health Organization, the number of people with hearing impairments is projected to rise dramatically, from approximately 1.1 billion today to 2.5 billion by 2050, underscoring the growing importance of understanding and addressing hearing-related issues.

The human ear is a complex organ divided into three main parts: the outer ear, the middle ear, and the inner ear. The outer ear consists of the auricle, the external auditory canal, and the outer layer of the tympanic membrane. The middle ear includes the tympanic membrane, the tympanic cavity, the auditory ossicles (the malleus, incus, and stapes), the Eustachian tube, and the external surface of the oval window. The inner ear comprises the cochlea, the semicircular canals, and the auditory nerve, which together facilitate the conversion of sound waves into neural signals.

This dissertation explores two key aspects of middle ear mechanics: the dynamics of an intact middle ear, including the relaxation phenomena in ligaments and

tendons, and the behavior of a middle ear that has been surgically implanted. The study delves into the viscoelastic properties of middle ear tissues, examining both Kelvin-Voigt and Maxwell models, which represent different types of viscoelastic behavior. These models are critical for understanding how the middle ear responds to sound waves and how its components, such as ligaments and tendons, contribute to its overall function.

Conducting experimental research on the human middle ear, whether on living subjects or temporal bones, presents significant challenges. Such studies require specialized procedures for handling human tissue, adherence to ethical guidelines, and access to rare specimens. As a result, numerical modeling emerges as a highly effective alternative for advancing our understanding of middle ear mechanics. Computational simulations allow researchers to explore new treatment methods, design innovative medical devices, and develop advanced prostheses for the middle ear without the limitations associated with physical experiments.

This dissertation is dedicated to a comprehensive analysis of middle ear dynamics, with a dual focus. First, it investigates the intact middle ear, emphasizing the role of relaxation phenomena in ligaments and tendons and their impact on sound conduction. Second, it examines the middle ear in implanted state, providing insights into how implants alter its mechanical behavior. By leveraging numerical models, this research aims to bridge the gap between theoretical understanding and practical applications. Through this work, the dissertation seeks to advance the fields of mechanical engineering, auditory and offer new insight for addressing hearing process.

2.1 Scientific problems and theses

This thesis addresses several critical scientific questions related to the mechanics of sound transfer in the human middle ear, particularly in the context of an implanted system. The aim is to deepen the understanding of how various phenomena and factors influence the performance and dynamics of sound transmission in both natural and implanted states. Specifically, the research seeks to answer the following key questions:

- How does the relaxation effect influence sound transfer through the middle ear?
- How does the bistability problem of the Floating Mass Transducer affect sound transmission in the implanted human middle ear?
- What is the impact of double excitation in an implanted middle ear, and how does the delay in the implant's electrical system influence sound transmission?

To address these questions, the following research hypotheses are formulated:

- **The relaxation phenomenon significantly affects stapes vibration.**
This hypothesis assumes that relaxation properties of the middle ear structures influence the vibrations of the stapes. Relaxation effects are critical to understanding the viscoelastic behavior of middle ear tissues and their role in sound transmission dynamics.
- **An implanted human middle ear can exhibit bistable behavior.**
This hypothesis focuses on possibility that the nonlinear mechanical properties of the FMT and its interaction with the ossicular chain may result in bistability. Such behavior could lead to multiple stable states of vibration, which has implications for the performance and optimization of middle ear implants.
- **Double excitation in the implanted human middle ear, as well as delay time in the implant's electrical system, affects stapes footplate vibration.**
This hypothesis shows the complex dynamics introduced by double excitation (sound at the eardrum and mechanical input from the FMT) and the role of delays caused by the implant's electrical components (in implant's control system). The study investigates how these factors influence the overall vibrational response of the stapes footplate.

2.2 Aim and scope

The primary scientific objective of this dissertation is to advance the understanding of the dynamic behavior of the human middle ear by exploring three significant

aspects: the role of ligaments and tendons relaxation in the intact middle ear, the nonlinear behavior of the middle ear with the FMT and the effect of double excitation on ossicular motion which come from the outer sound as well as implant excitation. In the case of an implanted middle ear, the eardrum remains a contributing factor to the motion of the ossicles, although its influence is reduced compared to an intact system. The peractical utility of the research is to identify and analyze periodic and irregular vibration of the stapes while incorporating the effects of relaxation, nonlinearity, and dual excitation. To achieve this, the dissertation includes:

- **Literature Review**

A comprehensive review of existing middle ear models and related research in the literature, identifying gaps and opportunities for further exploration.

- **Development of Mathematical and Numerical Models**

Formulating mathematical models to simulate the intact and implanted middle ear, including:

- A three-degree-of-freedom model for the intact middle ear.
- Two distinct five-degree-of-freedom models for the Implanted Middle Ear.

- **Experimental Studies**

Conducting experimental studies on human temporal bones to obtain intact middle ear characteristics, providing empirical data to validate the developed models and to verify IME model.

- **Conclusions and Implications**

Drawing final conclusions from the theoretical and experimental findings to advance the understanding of middle ear dynamics and inform future research and better implant design.

2.3 Novelty

This dissertation introduces several novel contributions to the field. One such innovation is the incorporation of ligaments tendons relaxation into viscoelastic mod-

els, including the Kelvin-Voigt and Maxwell models, to better capture the biomechanical behavior of the intact middle ear. Another key advancement is the investigation of the dynamics of double excitation in the implanted ear, where sound reaching the eardrum interacts with mechanical forces generated by the FMT. These interactions create complex vibrational patterns in the ossicular chain, which have not been thoroughly studied in existing literature. The findings of this research are expected to contribute to a more comprehensive understanding of middle ear mechanics and provide valuable insights into the design and optimization of the Implantable Middle Ear Hearing Devices. By bridging theoretical modeling with experimental data, this dissertation offers a fresh perspective on the interplay of mechanical and physiological factors in auditory biomechanics.

The review of the current state of knowledge

The human ear is a marvel of biological engineering, intricately designed to capture, amplify, and process sound. It is divided into three main sections: the outer ear, the middle ear, and the inner ear, each playing a distinct yet interconnected role in the auditory process. Sound waves first enter the outer ear, where they travel through the ear canal — a narrow, tube-like structure that channels the waves toward the TM. The eardrum, a thin and highly sensitive membrane stretched tightly across the entrance to the middle ear, vibrates in response to these sound waves. These vibration are then transmitted to the middle ear, initiating the movement of the bones that ultimately allows us to hear.

The middle ear is a small, air-filled cavity that houses three tiny bones known as the auditory ossicles: the malleus, the incus, and the stapes, presented in the Figure 3.1. These ossicles form a delicate yet highly efficient mechanical system that bridges the space between the eardrum and the inner ear. The malleus is directly connected to the eardrum, while the incus links the malleus to the stapes. The stapes, in turn, is attached to the oval window, a membrane-covered opening that leads to the fluid-filled inner ear. Together, these bones act as a sophisticated lever system, amplifying and transmitting sound vibration from the eardrum to the inner ear.

The ossicles are interconnected by two key joints: the incudo-malleal joint (between the malleus and incus) and the incudo-stapedial joint (between the in-

cus and stapes). This system is further supported by two small muscles: the tensor tympani, which is attached to the malleus, and the stapedius tympani which is connected to the stapes. These muscles play a crucial role in protecting the ear from damage caused by excessively loud sounds by damping the vibration of the ossicles. Additionally, the stapes is anchored to the bony walls of the oval window by the AL, forming the stapediovestibular joint. This entire structure is essential for the efficient transmission of vibration from the middle ear to the fluid-filled chambers of the inner ear.

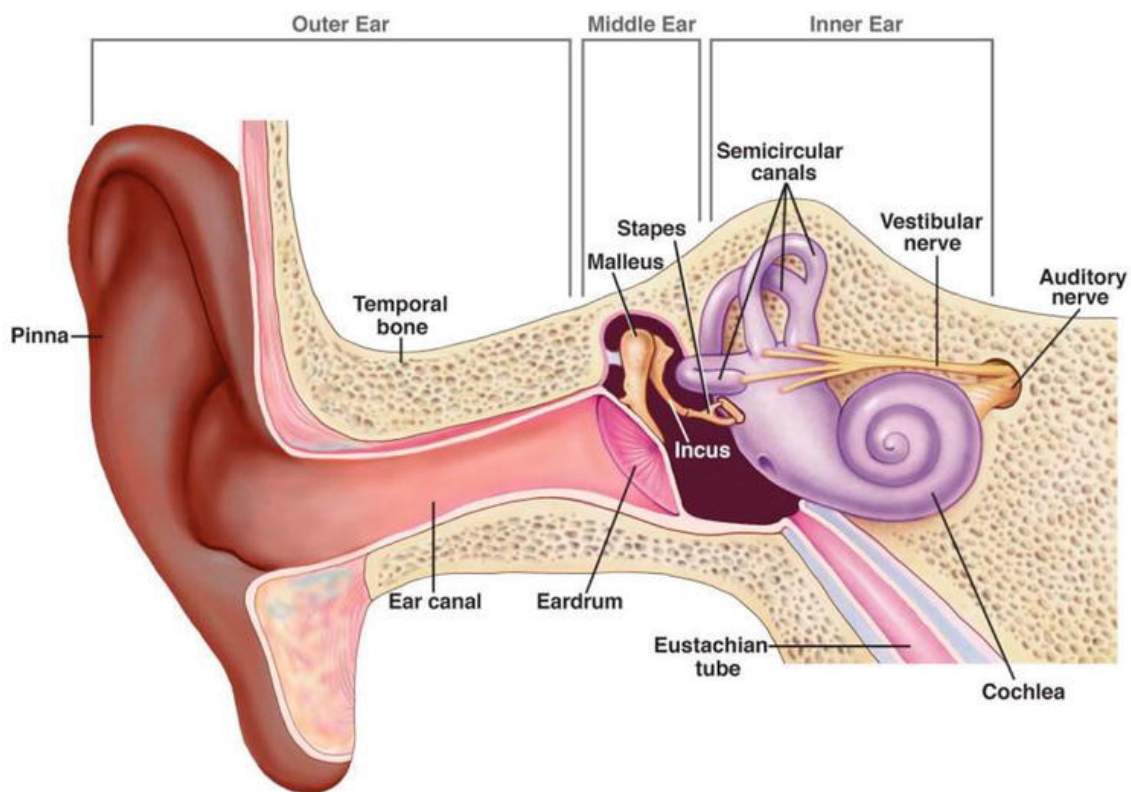


Figure 3.1. *Anatomical structure of the human hearing system. [source [84]]*

The inner ear, often referred to as the labyrinth, is a complex network of fluid-filled passages and cavities embedded deep within the temporal bone. It consists of two primary functional units: the vestibular system and the cochlea. The vestibular system, which includes the vestibule and semicircular canals, is responsible for maintaining balance and spatial orientation. The cochlea, on the other hand, is a snail-shaped structure that houses the sensory organs of hearing. These sensory organs, known as hair cells, are specialized nerve endings of the vestibulocochlear

nerve (the eighth cranial nerve). They convert mechanical vibration into electrical signals, which are then sent to the brain for interpretation.

The primary function of the middle ear is to act as an acoustic transformer, efficiently converting the vibration of the eardrum into movements of the stapes. This transformation is critical for ensuring that sound energy is effectively transferred from the air-filled middle ear to the fluid-filled inner ear. The middle ear achieves this through two key mechanisms:

- **Mechanical Lever Action:** The ossicles function as a mechanical lever, further amplifying the pressure exerted on the oval window. This lever action ensures that most of the sound energy entering the ear is transmitted to the inner ear with minimal loss.
- **Area Ratio Amplification:** The surface area of the eardrum is approximately 17 times larger than that of the oval window. This size difference increases the pressure (force per unit area) exerted on the oval window, effectively amplifying the sound energy.

Without these transformative mechanisms, only about $\frac{1}{1000}$ of the acoustic energy in air would reach the inner ear fluids, resulting in a significant energy loss of approximately 30 decibels (dB). This could seriously impair our ability to hear faint sounds and distinguish speech in noisy environments.

The human ear is a highly specialized and finely tuned system that relies on the precise coordination of its three main components — the outer, middle, and inner ear — to capture, amplify, and process sound. The middle ear, with its ossicular chain and amplifying mechanisms, plays a pivotal role in ensuring that sound energy is efficiently transmitted to the inner ear, where it is ultimately converted into the neural signals that allow us to perceive sound. Understanding the intricate workings of this system is not only fascinating but also essential for developing treatments for hearing disorders and improving auditory technologies.

3.1 Human Middle Ear

In this section, the biomechanics of the human middle ear—a critical system responsible for efficient sound transmission—is examined in detail. The middle

ear functions as an impedance - matching mechanism, converting airborne sound waves into mechanical vibrations that are then transferred to the inner ear. Each subsection focuses on a key anatomical structure involved in this process: the tympanic membrane (eardrum), which captures sound waves, the malleus, incus, and stapes, which form the ossicular chain and amplify these vibrations. Together, these components ensure precise and effective auditory signal transmission.

3.1.1 TM

The tympanic membrane, commonly known as the eardrum, marks the boundary between the external ear canal and the middle ear, serving as the primary medium for sound transmission. It can be directly examined using an otoscope, a device equipped with a light source and magnifier. Structurally, the tympanic membrane has a conical shape, resembling a horn, which is believed to function as a waveguide, enhancing the transmission of high-frequency sounds [30, 32]. This pronounced conical form, especially distinct in mammals, is thought to result from the inward pull exerted by middle ear structures, a concept originally proposed by Helmholtz [39]. The membrane's unique morphology may also contribute to the broader auditory bandwidth observed in mammals compared to other terrestrial vertebrates [32]). The tympanic membrane's conical shape is reinforced by a complex fibrous network within its intermediate layer. The central region is primarily composed of radial fibers, while circumferential fibers become denser toward the periphery [30]. These fibers, made of collagen types I, II, and III [90], provide structural integrity. As shown in Figure 3.2, this specific fiber arrangement plays a crucial role in determining the membrane's mechanical properties.

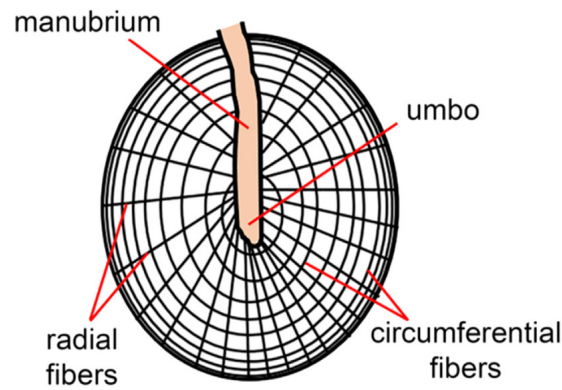


Figure 3.2. *The tympanic membrane characterized by a network of superficial radial fibers and deeper circumferential fibers. [source [95]]*

The mechanical stiffness of the tympanic membrane has been widely studied using various experimental techniques. Von Békésy in [97] pioneered research with static bending tests using a calibrated probe, reporting a Young's modulus of 20 MPa. Later studies utilized longitudinal tensile tests [53, 26, 14] and nanoindentation - method involves pressing a small, hard indenter into the tissue surface and measuring the applied force and resulting deformation on tissue sections [43, 18] to assess stiffness. However, reported values vary widely, ranging from 2.1 to 300 MPa for whole membranes [29, 1, 73] and from 0.4 to 58.9 MPa in longitudinal tensile tests. This variability arises due to differences in testing methods, specimen preparation, inter-specimen variations, and the heterogeneous nature of the tympanic membrane, including the specific location from which tissue samples were taken [73]. Despite these discrepancies, research consistently indicates that the Young's modulus of the tympanic membrane increases with applied stress and frequency. Studies in both quasi-static [14] and dynamic [109] conditions confirm this trend, underscoring the membrane's adaptive mechanical behavior [61]. These findings highlight the tympanic membrane's pivotal role in auditory function, demonstrating its complex and frequency-dependent mechanical properties.

3.1.2 Malleus

Figure 3.3A illustrates the human malleus, characterized by a thick, rounded head and a thinner manubrium. The neck of the malleus, located between the head and manubrium, forms the surface for the incudomalleolar joint. In [4] it was assumed

that this joint functions as a rigid block, with no articular movement between the incus and malleus. Below the neck, two processes emerge: the long process (anterior process or processus Folianus) and the short process (lateral process or processus brevis) [40]. The long process, covered by ligaments, points anteriorly, while the short process attaches to the tympanic membrane. De Greef [23] identified 13 ligaments in the human middle ear, seven of which attach to the malleus: the anterior, medial anterior, superior anterior, lateral, posterior, anterior superior, and posterior superior malleolar ligaments. The AML and PML stabilize the malleus, even in the absence of other ossicles. Mechanical testing revealed a Young's modulus of 3.8 ± 0.5 GPa for the human malleus and incus [89], while micro-indentation tests on rabbit malleus yielded stiffness values of 16 ± 3 GPa for the head and 15.6 GPa for the neck [88]. The AML exhibits a Young's modulus of around 6 MPa under 0.5 MPa stress [15, 16]. The AML and PML also serve as rotational axes for the malleus, with the A-SML and P-SML preventing excessive rotation [40, 23]. The malleus attaches to the tympanic membrane via the manubrium, extending from the umbo to the lateral process as shown in Figure 3.3A.

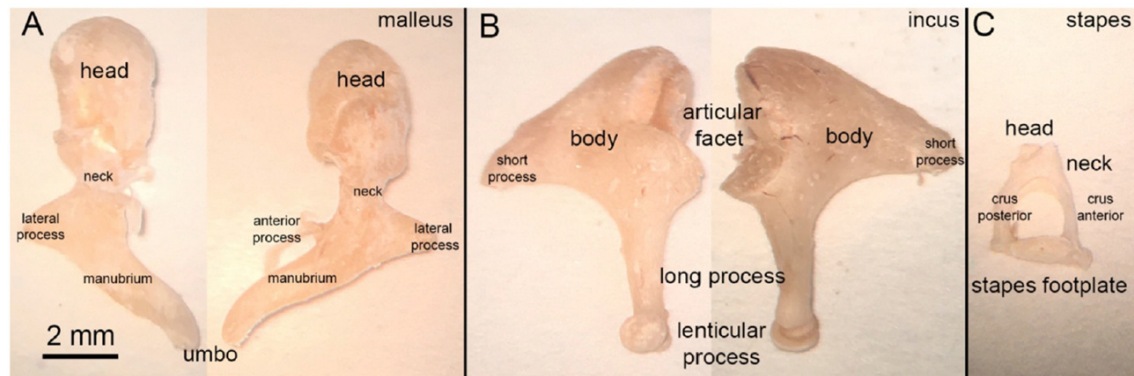


Figure 3.3. Human middle ear - ossicle chain: A) Malleus, B) Incus, C) Stapes. [source [95]]

3.1.3 Incus

Figure 3.3B depicts the incus, featuring a short process, a long process, and a lenticular process. The lenticular process connects to the stapes at the incudostapedial joint, while the incudomalleolar joint, a biaxial saddle-shaped structure, allows movement in two planes [72]. Lauxmann [57] measured rupture forces in humans, averaging 894 mN in the anterior-posterior direction and 695 mN in the lateral-

medial direction. Micro-indentation tests on rabbit incus revealed Young's moduli of 16.8 ± 3 GPa for the body and 17.1 GPa for the long and short process [88]. Spatial distribution patterns of chloride, calcium, potassium, and zinc in gerbill incus suggest active ossification areas [5]. In humans, the synovial-fluid-filled gap between the malleus and incus ranges from 40 to 320 μm [72, 87]. The incus is supported by four ligaments: the medial incudomalleolar ligament and the posterior, medial, and lateral incudal ligaments. The PIL anchors the incus to the posterior wall of the middle ear cavity.

3.1.4 Stapes

The stapes consists of a head, neck, anterior and posterior crus, and a footplate that inserts into the oval window of the cochlea (Figure 3.3C). The AL, a viscoelastic structure with nonlinear properties, seals the stapes footplate into the oval window [34]. Mechanical properties of the AL vary, with Young's modulus ranging from 0.01 MPa under uniaxial stress [64] to 1.1×10^{-3} GPa using atomic force microscopy [56]. The AL forms the stapediovestibular joint, bridging the gap between the stapes footplate and the cochlea. The ISJ, the smallest joint in the body, is a synovial joint with cartilage and synovial fluid [67, 101]. Mechanical testing of the human ISJ revealed a Young's modulus of 14.1 GPa for the incus and stapes head, 1.0×10^{-2} GPa for the cartilage, and a bulk modulus of 2.2 GPa for the synovial fluid [33, 111]. The ISJ exhibits viscoelastic behavior with a nonlinear stress-strain relationship [111].

3.2 Human Ear Models

The human ear is a complex organ responsible for hearing and balance. To better understand its structure and function, researchers have developed anatomical models that aid in studying ear mechanics, diagnosing disorders, and designing treatments. These models are typically categorized into three regions: outer, middle, and inner ear—each playing a distinct role in sound transmission and balance. Middle ear models focus on the ossicular chain (malleus, incus, and stapes), which amplifies and transmits sound vibrations to the inner ear. This section explores

the development, applications, and significance of different types of ear models in advancing auditory science and medical interventions.

3.2.1 Outer Ear

The ear canal has been extensively studied using lumped parameter models to understand its influence on sound transmission to the tympanic membrane. At low frequencies, the ear canal can be approximated as a rigid-walled cavity, defined primarily by its volume. In electrical circuit models, this effect can either be adjusted for in experimental data [115] or represented explicitly by a single capacitance. As frequencies increase, the wavelength becomes comparable to the length of the canal, leading to the formation of standing-wave patterns. A simple mass-spring system can be used for simplified systems [68, 69], or the canal can be treated as a uniform transmission line [104]. To account for the non-uniform transverse dimensions of the canal and the distributed acoustical impedance of the tympanic membrane, a one-dimensional modified horn equation has been employed [52, 93] and is a type of mathematical model used to describe sound wave propagation in a tube with a varying cross-sectional area, taking into account factors like non-uniformity and non-linearity. Numerous experimental studies have measured the ear canal's impedance, reflection coefficient [51, 99], sound power per unit cross-sectional area [28], and the pressure levels at the ear canal entrance for various incident sound angles [83, 91] utilized the modified horn equation to predict pressure distribution along the canal's curvature. More recently, Kahana and Nelson [49, 50] created a precise 3D mesh of the head, auricle, and external ear geometry using laser scanning technology. They derived the frequency response of their model by solving the Helmholtz equation with the boundary element method. Additionally, Stinson and Daigle [92] analyzed the sound field along the central axis and cross-sections of a detailed human ear canal model, employing both an analytic horn equation and a boundary element method to predict sound pressure magnitudes at different frequencies and along the canal's curvature. These approaches collectively provide a comprehensive understanding of the ear canal's role in sound conduction.

3.2.2 Middle Ear

The HME plays a crucial role in the peripheral auditory system by efficiently transmitting acoustic energy from the outer ear to the inner ear. This transmission is facilitated through impedance matching between the low-impedance air in the ear canal and the high-impedance cochlear fluid. While the general function of the HME is well understood, several aspects of its dynamic behavior remain subjects of ongoing research. These include the impact of varying the TM material properties on sound transmission and the effects of middle-ear muscle contraction during the acoustic reflex, which are critical for predicting potential hearing damage from noise exposure [66, 48, 102]. A deeper understanding of the HME's dynamic behavior is essential for analyzing different auditory conditions, including conductive hearing disorders and their impact on sound transmission [62]. This knowledge is also vital for assessing the outcomes of corrective surgeries and the performance of passive and active prostheses [44, 27], as well as evaluating the functionality of implantable hearing aids [17, 10, 9]. Both experimental methods and numerical modeling have been employed to study the mechanics of the HME. Many experimental studies focus on characterizing sound transmission by analyzing the FRF of stapes footplate velocity in relation to TM sound pressure [100, 74, 2]. Traditionally, these studies assume that stapes movement occurs primarily in a lateral-medial direction, known as "piston-like" motion [38, 31, 45]. However, the complexity of TM motion and the ossicular chain dynamics introduces additional translational and rotational movements, particularly at higher frequencies [38, 25, 86, 27]. Consequently, stapes motion deviates from the simple piston-like assumption. Velocity measurements of specific points on the stapes footplate reveal rotational movements in the stapedio-vestibular joint, often described as "rocking" or "tilting" motion. However, experimental results obtained from OC velocity measurements exhibit notable discrepancies, likely due to challenges in accurately assessing movement direction, velocity magnitude, and laser beam alignment. To address the limitations of experimental procedures, computational models of the HME have been developed [24, 71].

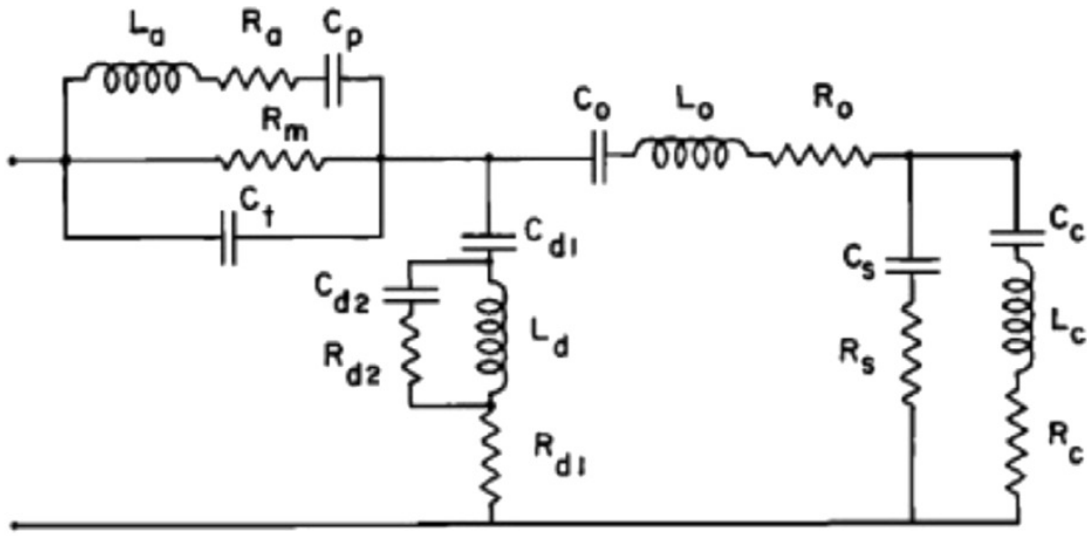


Figure 3.4. Middle Ear represented as electric circuit. [source [116]]

Lumped-parameter models are among the simplest approaches to modeling the middle ear, representing its function through electrical equivalent circuits. In these models, the electrical components are defined by their acoustic impedance properties: inductors correspond to acoustic mass, capacitors represent acoustic compliance, and resistors symbolize acoustic resistance. These models provide a foundational framework for understanding the mechanical and acoustic behavior of the middle ear. One of the earliest and most influential lumped-parameter models was developed by Zwislocki [116]. This model describes the acoustic input impedance of the middle ear and includes components representing key anatomical structures such as: the middle ear cavities, the tympanic membrane, the ossicular chain, and the cochlea. Additionally, the model incorporates two shunts to account for sound energy that is reflected back into the ear canal without reaching the cochlea, typically due to non-ossicular coupling or impedance discontinuities (Figure 3.4). Zwislocki's model has been widely cited and provides a reasonable approximation of the acoustic input impedance of the intact human ear up to approximately 2 kHz [116]. Building on Zwislocki's work, Kringlebotn [55] expanded the model by including the ear canal as a rigid-walled tube, modeled as a lossy transmission line terminated by the middle ear's load impedance. Kringlebotn also introduced additional circuit elements to represent the elastic attachment of the eardrum to the walls of the ear canal. This refinement improved the model's accuracy and ex-

tended its applicability. Withnell and Gowdy [105] validated Kringlebotn's model against experimental data from six human ears, demonstrating its effectiveness in describing middle ear behavior up to 6 kHz, which was the upper frequency limit of their study. Despite their utility, lumped-parameter models have inherent limitations, particularly in their restricted frequency range of operation. They are unable to fully capture the broad frequency range of human hearing, which extends well beyond the capabilities of these simplified representations. To address this limitation, researchers have turned to more advanced modeling techniques, such as incorporating transmission lines, which better account for wave propagation effects and higher-frequency behavior. Notable contributions in this area include the works [70, 66, 32], which have developed models that extend the frequency range and improve the accuracy of middle ear simulations.

Finite element models, which offer a more comprehensive representation of ME mechanics, are widely used due to their versatility. These models typically present results in terms of TM surface displacement [12, 22], FRF of stapes velocity, TM sound pressure [54, 35], or modal shapes [42, 107]. Various analytical approaches have been employed with FE models to investigate the effects of different material properties [113, 54, 112, 8], boundary conditions [110, 23, 60], and structural dynamics on ME behavior. However, the high number of DOFs in FE models makes analyzing OC movement and its key influencing parameters complex. Additionally, implementing FE models requires substantial computational resources. As an alternative, multi-body models provide a balance between complexity and computational efficiency by integrating flexible structural analysis with rigid body dynamics. Also referred to as three-dimensional LP models, MB models typically treat ossicles as rigid bodies [27], significantly reducing DOFs compared to FE models. The incorporation of kinematic constraints [82] allows for simplified three-dimensional analysis of ME dynamics. While MB modeling is relatively recent, its applications include studying prosthesis and actuator implantation [27, 7], optimizing model parameters to align with experimental data [96], investigating joint flexibility [44], and implementing model reduction techniques [46]. Due to their low computational cost, MB models offer significant potential for further exploration of ME dynamics and movement analysis.

As illustrated in Figure 3.5, the human middle ear is a mechanically intricate system that efficiently transmits sound vibrations from the external environment to the inner ear. The TM establishes its connection to the malleus via the handle of the malleus, forming the primary interface for acoustic signal conduction. The malleus is further anchored to the temporal bone through several ligamentous attachments, notably the lateral malleal ligament, the AML, and the tensor tympani tendon, which collectively stabilize its position. Located in the epitympanic recess, the head of the malleus articulates with the body of the incus via the IMJ. The incus, structurally composed of a body, short process, and long process, forms a critical intermediary in the ossicular chain. The short process is anchored to the posterior wall of the middle ear, while the long process projects toward the stapes and forms the ISJ, enabling sound transmission to the final ossicle. The incus is further stabilized by the superior malleal ligament and the PIL, which secure it to the surrounding bone, ensuring functional integrity during vibratory motion. The stapes, the smallest bone in the human body, completes the ossicular chain by transmitting mechanical energy from the incus to the oval window of the cochlea. The footplate of the stapes is connected to the oval window via the AL, and to the temporal bone via the stapedius tendon. This configuration allows the stapes to efficiently deliver acoustic energy into the fluid-filled inner ear. The middle ear also contains two small but critical muscles: the tensor tympani and the stapedius. These muscles serve a protective function; the stapedius muscle, in particular, contracts reflexively in response to loud acoustic stimuli. This contraction reduces the amplitude of ossicular vibrations, thereby attenuating sound transmission to the cochlea—a mechanism known as the acoustic reflex. However, for the purpose of this study's mechanical modeling, the acoustic reflex has been excluded to simplify the analysis.

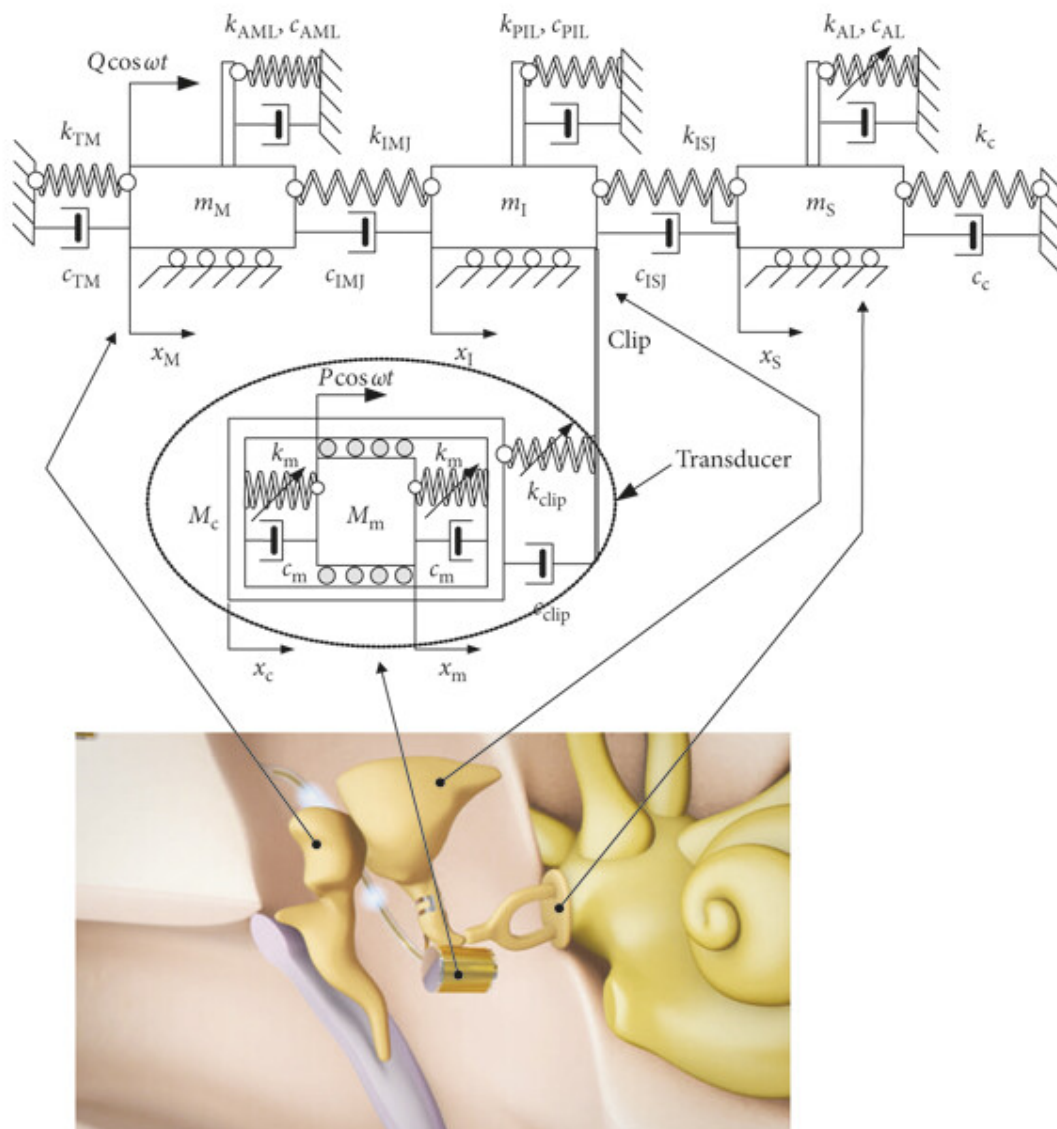


Figure 3.5. 5 DOF LPM featuring FMT without electro-mechanical coupling (top view), implanted human middle ear view (bottom view). [source [75]]

Behavior of the middle ear is represented using a lumped parameter model in this case from [75], wherein the anatomical components are idealized as discrete masses, springs (k), and dampers (c). Each of the three ossicles—the malleus (m_M), incus (m_I), and stapes (m_S) - is modeled as a rigid mass capable of horizontal motion on a fixed base. The AL, which connects the stapes to the cochlea, exhibits nonlinear (cubic) stiffness properties, denoted as k_{AL3} . The FMT is a key component in implantable hearing devices. It consists of a floating mass - magnet (M_m) and a metal case (M_c), which together constitute a dynamic structure sus-

pended by silicon springs (k_m) and dampers (c_m). When exposed to an alternating electromagnetic field, the magnet within the FMT oscillates, driven by an external force characterized by an amplitude P and frequency ω , as illustrated in Figure 3.5a. The FMT is mechanically coupled to the long process of the incus via a clip (coupler), which introduces additional damping and stiffness to the system. The coupler's mechanical properties are defined by linear and nonlinear coefficients: c_{CLIP} (damping), and k_{CLIP} (stiffness). Notably, the coupler's stiffness is assumed to be nonlinear and patient-specific, allowing for fine-tuning during clinical implementation. This adaptability is vital not only for optimizing mechanical performance but also from a maintenance and long-term reliability perspective, ensuring that the implant functions effectively over time.

In this study, the comparison of numerical models—both against each other and against experimental data—is conducted primarily through FRF analysis. An FRF curve characterizes how a system responds to a given input across a range of frequencies, making it an essential tool for evaluating the dynamic behavior of biomechanical systems such as the middle ear. The horizontal axis of an FRF plot consistently represents frequency, indicating the input signal's frequency at which the response is evaluated. The vertical axis, on the other hand, may display various response parameters, such as displacement, velocity, or acceleration, depending on the nature of the simulation or experiment. In many cases throughout this work, these mechanical responses are related to sound pressure levels within the middle ear, providing a more physiologically relevant metric for assessing hearing system performance. To ensure consistency and physiological relevance in the interpretation of FRF curves, this study references the ASTM F2504 standard, which defines acceptable frequency response ranges for the HME models. This standard, based on the foundational work of Rosowski [74], provides guidelines for validating middle ear simulations by setting benchmark criteria for response magnitude and frequency behavior. Adhering to this standard allows for objective evaluation of model accuracy and helps ensure that the simulated responses fall within a clinically and experimentally verified range.

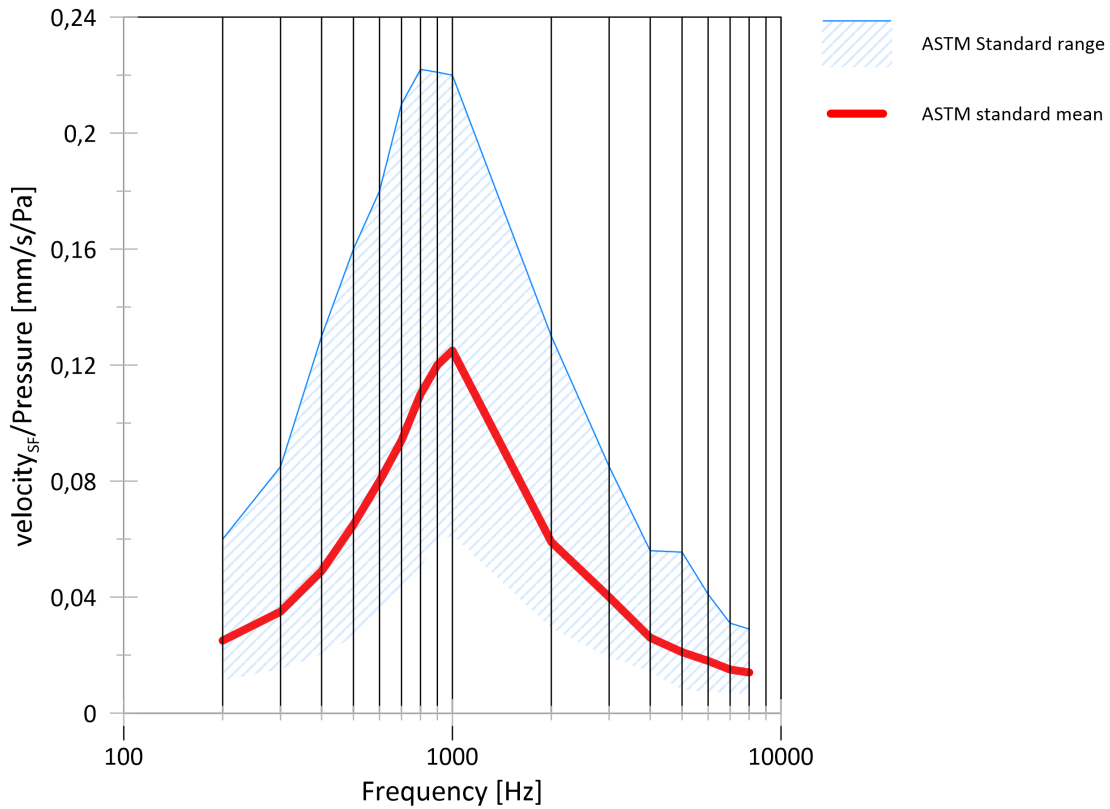


Figure 3.6. The FRF plot taken from ASTM F2504 standard [3] velocity produced by 94 dB sound pressure level.

Figure 3.6 presents the benchmark results defined by the ASTM F2504 standard, which serves as a reference for evaluating the frequency response of the human middle ear under normal (intact) conditions. This standard is derived from an extensive compilation of experimental data gathered from multiple independent studies, as collected and analyzed by Rosowski et al. in [74]. The figure illustrates the mean FRF curve based on these empirical measurements, along with corresponding upper and lower deviation bounds, which represent the expected variability across different specimens and experimental setups. These bounds define an acceptable range within which FRF responses are considered physiologically accurate for an Intact Ossicular Chain. It also shows the resonance peak around 1kHz. According to the standard, a given measurement or simulation is considered valid if its FRF curve lies entirely or mainly within this range. This provides a clear, quantitative criterion for assessing the physiological realism of experimental data or computational models. Furthermore, this standard allows for direct comparison

between model predictions and empirical observations, enabling researchers to assess the accuracy and fidelity of their simulations. By comparing model-generated FRF curves to the ASTM F2504-defined range, one can determine whether the model appropriately captures the dynamic behavior of the middle ear across the relevant frequency spectrum.

3.2.3 Inner Ear

Physical models of the cochlea have been developed to study aspects of hearing function that are difficult to investigate *in vivo* due to the cochlea's inaccessibility. Historically, the development of these models can be divided into two distinct periods, each characterized by unique challenges and advancements. In the early stages, the fabrication and actuation of such small devices posed significant difficulties, resulting in mechanical cochlea that were typically one or two orders of magnitude larger than the human cochlea. Moreover, the material properties of these early models were often not properly scaled, limiting their accuracy. For example, Von Békésy [98] constructed a mechanical cochlea to study traveling waves, while Tonndorf [94] used a similar design to investigate cochlear beats. Chadwick and Adler [11] developed a mechanical model to test their theories of cochlear function. Later, Lechner [58] introduced a cochlear model incorporating a Polyvinylidene Fluoride bending transducer mounted on beams within an artificial basilar membrane to generate electrical outputs from vibrational activity. Lechner also integrated actuators to simulate the cochlea's active response. Around the same time, Zhou et al. [114] proposed the first life-sized physical model of the basilar membrane, though it lacked precise control over membrane thickness and used a fluid viscosity 20 times higher than that of perilymph.

The second period began in the mid-1990s with the advent of improved micromachining technology. Haronian and MacDonald [37] introduced a micro-electro-mechanical frequency signature sensor, consisting of an array of silicon beams with gradually varying lengths mounted on a core backbone to simulate fluid channel vibration. In 1999, Manoussaki and Chadwick [65] used both straight and coiled cochlea models to study the influence of geometry on fluid-cochlear partition coupling. They found that coiling reduces inertial impedance to fluid motion, decreases basilar membrane pressure with reduced coil pitch, and maximizes

impedance reduction at the cochlear apex. Lim et al. [59] created a mechanical cochlea using an isotropic film on a slotted silicon wafer surrounded by fluid channels, demonstrating traveling wave propagation but with irregular magnitude responses. Hemmert et al. [41] developed a fluid-filled MEMS-based cochlea with an epoxy-based photoresist BM, using impulse responses to confirm traveling waves. Wittbrodt et al. [106] advanced Lim's work by depositing aluminum fibers on a soft membrane to create an anisotropic ABM. White and Grosh [103] improved their MEMS cochlea by using silicon-nitride beams to reduce residual tension, though the stiffness of these materials still differed significantly from biological tissues, leading to abnormal magnitude responses.

More recently, Jang et al. [47] proposed a MEMS-based piezoelectric ABM composed of a cantilever array integrated with a signal processor and intracochlear electrode array. When exposed to acoustic sounds, the cantilevers generated piezoelectric outputs, which were converted into electrical signals to stimulate auditory neurons in deafened guinea pigs, eliciting evoked auditory brain responses. While this design holds promise for low-cost cochlear implants, challenges related to the size of the signal processor and energy supply have prevented direct implantation. Despite recent advancements in microfabrication, physical cochlear models still struggle to accurately replicate cochlear mechanics compared to numerical models, which benefit from increasing computational power. This gap highlights the ongoing need for innovation in both material science and engineering to better mimic the intricate functionality of the biological cochlea.

Models of the Human Middle Ear

Lumped Parameter Models are simplified representations of complex mechanical systems that consolidate spatially distributed properties into discrete elements. Those models are widely used in mechanics and solid dynamics. They make the analysis easier by approximating their behavior through a finite number of degrees of freedom. This abstraction is particularly valuable in situations where detailed spatial modeling is impractical or computationally expensive. In the context of mechanics, LPMs translate physical phenomena such as mass, stiffness, and damping into discrete components—often represented as mass points, springs, and dashpots, respectively. This reductionist approach assumes that the spatial variations of these parameters are negligible or can be averaged effectively, allowing engineers and researchers to focus on key dynamic and static responses without the complexity of distributed modeling. In solid dynamics, LPMs play a critical role in studying vibrations, dynamic responses, and impact phenomena. By discretizing a continuous solid into a network of lumped elements, the equations of motion become solvable using classical techniques or numerical methods, enabling predictions of system behavior under various conditions. These models are particularly useful in applications like structural dynamics, machine design, and biomechanics, where understanding the interplay of forces, deformations, and energy dissipation is essential. The utility of Lumped Parameter Models lies in their balance between simplicity and accuracy. While they sacrifice some fidelity compared to distributed

parameter models, they provide valuable insights and are computationally efficient, making them indispensable tools in engineering analysis and design. This introduction explores the fundamental principles, applications, and limitations of LPMs in the fields of mechanics and solid dynamics, highlighting their relevance in both academic research and industrial practice.

4.1 Intact Human Middle Ear

In the HME, the malleus is affixed to the tympanic membrane through the malleus handle and is anchored to the temporal bone by the lateral malleal ligament, tensor tympani tendon, and anterior malleal ligament Figure 4.1. The head of the malleus resides in the epitympanic recess, where it articulates with the adjacent auditory ossicle, the incus, via the IMJ. The incus comprises a body and two processes: the body links to the malleus, the short process attaches to the posterior wall of the ME, and the long process connects the stapes to the ISJ. Tying the incus to the bone are the superior malleal and posterior incudal ligaments. The stapes, the tiniest bone in the human body, bridges the gap between the incus and the oval window of the inner ear. The stapes' footplate is tethered to the oval window by way of the AL and to the temporal bone via the stapedius tendon. Two protective muscles in the middle ear play vital roles: the tensor tympani and the stapedius muscle. When exposed to loud noises, the stapedius muscle contracts, damping the vibrations of the ossicles and reducing sound transmission to the inner ear. This phenomenon, known as the acoustic reflex, is not considered in this study when modeling the sound transmission process. Furthermore, only the components directly responsible for sound transmission within the audible range are taken into account. Therefore, the proposed models of the HME consist of interconnected masses using springs (k) and dampers (c), configured in the Maxwell or Kelvin-Voight (K-V) viscoelasticity type, depending on the assumed models depicted in Figures 4.1 and 4.2. The ossicles are represented by three masses: the malleus (m_M), the incus (m_I), and the stapes (m_S). These lumped masses (m_M , m_I , m_S) can move horizontally on a base, which excludes rotational movement but has no impact on the primary vibration modes. The ossicles are interconnected via the IMJ and ISJ joints and attached to the temporal bone by the AML,

PIL, and AL ligaments, represented by the viscoelastic material model comprising dampers and springs in series or parallel connections. This results in the modified Maxwell model, also known as the Zener model (Figure 4.1, offering a more realistic representation of tissue behavior compared to the commonly used K-V model (Figure 4.2). The K-V model is employed for describing the damping and stiffness properties of the cochlea (c_c and k_c) and the tympanic membrane (c_{TM} , k_{TM}) in both Figures 4.1 and 4.2. Additionally, the AL exhibits nonlinear (cubic) stiffness characteristics denoted by k_{AL3} . The ear is stimulated by an external signal, $Q\cos(\omega t)$, acting upon the malleus through the tympanic membrane. In Figure 4.1, the model highlights the most crucial joints and ligaments for sound transmission, indicated in bold. These elements are incorporated into the study's modeling approach. Moreover joints and ligaments show the relaxation effect which is reported in [80, 20, 21]. In both models (Figure 4.1 and 4.2) the relaxation phenomenon is considered using a modified Young's modulus (E) of the joints and ligaments having the form:

$$E = E_0 + E_1 e^{-t/\tilde{t}_1} \quad (4.1)$$

where E_0 and E_1 denote the final and the changeable component of the elastic (Young's) modulus of the tissue. Then, the preliminary modulus is $E_0 + E_1$ while $\tilde{t}_1$ denotes the relaxation time; therefore, a new stiffness, k_r is defined as:

$$k_r = k_0 \left(1 + \frac{E_1}{E_0} e^{-t/\tilde{t}_1} \right) \quad (4.2)$$

where, $k_0 = \frac{AE_0}{L}$, A and L denote the cross section and the length of the joint or ligament, respectively.

In this chapter as well as in the results the following notations are used:

- Model 1 - model with Maxwell type of viscoelasticity,
- Model 2 - model with Kelvin-Voight type of viscoelasticity,
- Model 3 - model with Kelvin-Voight type of viscoelasticity and relaxation effect.

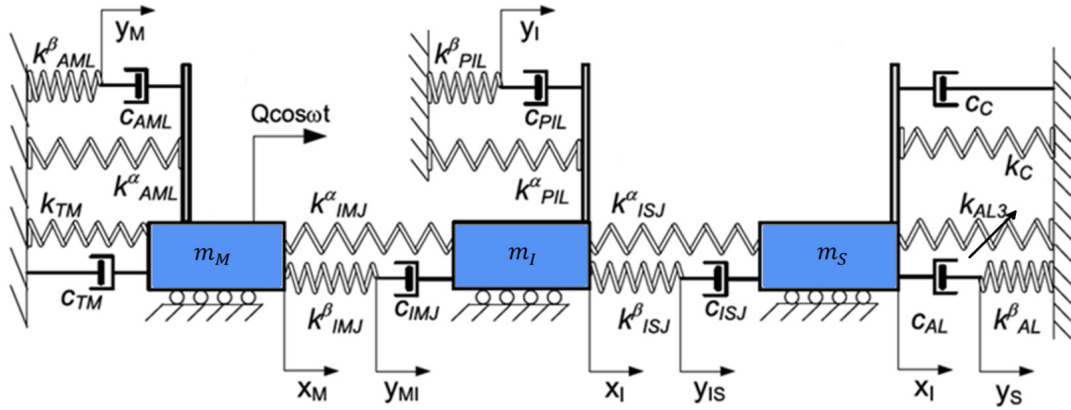


Figure 4.1. The 3-DOF model of the human middle ear incorporating Maxwell-type viscoelastic properties.

In the model illustrated in Figure 4.1, the overall stiffness (k) of the joints and ligaments is split into two components, k^α and k^β , with k^β (see Figure 4.1) being in series with a dashpot. The complete stiffness (k) can be expressed by the following equation:

$$k = k^\alpha + k^\beta \quad (4.3)$$

To control both components of the stiffness coefficients α and β , the following relationship is introduced:

$$k^\alpha = \alpha k, \quad k^\beta = \beta k, \quad \alpha + \beta = 1 \quad (4.4)$$

This also allows for transitioning to the pure K-V model by removing k^β . The middle ear with K-V viscoelastic properties is depicted in Figure 4.2.

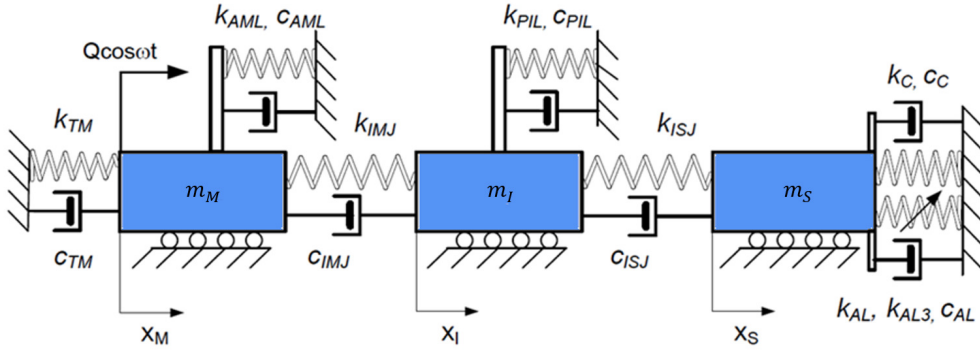


Figure 4.2. The 3-DOF model of the human middle ear employing Kelvin-Voigt-type viscoelasticity for the analysis of systems with and without relaxation effects.

The governing differential equations of motion for the three models are formulated using the second-type Lagrange equation, expressed as:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} + \frac{\partial V}{\partial x_i} + \frac{\partial \Phi}{\partial \dot{x}_i} = Q_i. \quad (4.5)$$

In all following equations subscripts are in accordance to the notation from Table 1. The potential energy (V) and the Rayleigh function of dissipation (Φ) are established for each model, with the kinetic energy (T) being uniform across all the models and represented as follows:

$$T = \frac{1}{2} (m_M \dot{x}_M^2 + m_I \dot{x}_I^2 + m_S \dot{x}_S^2) \quad (4.6)$$

The middle ear model incorporating Kelvin-Voigt viscoelasticity is distinguished by its potential energy, which is expressed as follows:

$$V = \frac{1}{2} (k_{TM} x_M^2 + k_{AML} x_M^2 + k_C x_S^2 + k_{AL} x_S^2 + k_{PIL} x_I^2 + k_{IMJ} (x_M - x_I)^2 + k_{ISJ} (x_I - x_S)^2) + \frac{1}{4} k_{AL3} x_S^4 \quad (4.7)$$

where:

x - displacement of the structure in the K-V system.

Meanwhile, the Rayleigh function of dissipation (Φ) is defined as:

$$\Phi = \frac{1}{2}(c_{TM}\dot{x}_M^2 + c_{AML}\dot{x}_M^2 + c_C\dot{x}_S^2 + c_{AL}\dot{x}_S^2 + c_{PIL}\dot{x}_I^2 + c_{IMJ}(\dot{x}_M - \dot{x}_I)^2 + c_{ISJ}(\dot{x}_I - \dot{x}_S)^2) \quad (4.8)$$

where:

$\dot{x}$ - velocity of the structure in the K-V system.

Differential equations of motion are derived by substituting Equations 4.6, 4.7 and 4.8 into Equation 4.5, resulting in:

$$\begin{aligned} \ddot{x}_M m_M + \tilde{k}_{11}x_M + \tilde{k}_{12}x_I + \tilde{c}_{11}\dot{x}_M + \tilde{c}_{12}\dot{x}_I &= Q_0 \cos(\omega t) \\ \ddot{x}_I m_I + \tilde{k}_{21}x_M + \tilde{k}_{22}x_I + \tilde{k}_{23}x_S + \tilde{c}_{21}\dot{x}_M + \tilde{c}_{22}\dot{x}_I &= 0 \\ \ddot{x}_S m_S + \tilde{k}_{32}x_I + \tilde{k}_{33}x_S + \tilde{c}_{32}\dot{x}_I + \tilde{c}_{33}\dot{x}_S + \tilde{k}_{AL3}x_S^3 &= 0 \end{aligned} \quad (4.9)$$

where

$$\begin{aligned} \tilde{k}_{11} &= k_{TM} + k_{AML} + k_{IMJ}, \quad \tilde{k}_{12} = \tilde{k}_{21}^M = -k_{IMJ}, \\ \tilde{k}_{22} &= k_{PIL} + k_{IMJ} + k_{ISJ}, \quad \tilde{k}_{23} = k_{32} = -k_{ISJ}, \quad \tilde{k}_{33} = k_{AL} + k_{ISJ} + k_C \\ \tilde{c}_{11} &= c_{TM} + c_{AML} + c_{IMJ}, \quad \tilde{c}_{12} = \tilde{c}_{21} = -c_{IMJ}, \\ \tilde{c}_{22} &= c_{PIL} + c_{IMJ} + c_{ISJ}, \quad \tilde{c}_{23} = c_{32} = -c_{ISJ}, \quad \tilde{c}_{33} = c_{AL} + c_{ISJ} + c_C \end{aligned} \quad (4.10)$$

By defining the dimensionless time τ , frequency Ω , and coordinates x_1 to x_3 , the equations of motion are transformed into the form presented in Equation (4.10). The new dimensionless coefficients and coordinates are specified as follows:

$$\begin{aligned} k_{ij} &= \tilde{k}_{ij}/(m_m \omega_0^2), \quad c_{ij} = \tilde{c}_{ij}/(m_m \omega_0), \quad \gamma_3 = k_{AL3}x_{st}^2/(m_m \omega_0^2), \\ i, j &= 1 \dots 3 \quad q = Q/(m_m \omega_0^2 x_{st}), \quad \tau = \omega_0 t, \quad x_1 = x_M/x_{st}, \\ x_2 &= x_I/x_{st}, \quad x_3 = x_S/x_{st}. \end{aligned} \quad (4.11)$$

where:

$$\omega_0 = \sqrt{k_{AML}/m_M}, \quad x_{st} = 1mm$$

Final, dimensionless form of the differential equations of motion, that are used to build numerical Kelvin-Voight type model, are shown below:

$$\begin{aligned}\ddot{x}_1 m_1 + k_{11r} x_M + k_{12r} x_I + c_{11} \dot{x}_M + c_{12} \dot{x}_I &= q_0 \cos(\omega t) \\ \ddot{x}_2 m_2 + k_{21r} x_M + k_{22r} x_I + k_{23r} x_S + c_{21} \dot{x}_M + c_{22} \dot{x}_I &= 0 \\ \ddot{x}_3 m_3 + k_{32r} x_I + k_{33r} x_S + c_{32} \dot{x}_I + c_{33} \dot{x}_S + \gamma_3 x_S^3 &= 0\end{aligned}\quad (4.12)$$

The middle ear model incorporating Maxwell viscoelasticity is distinguished by its potential energy, which is expressed as follows:

$$\begin{aligned}V = \frac{1}{2} (k_{TM} x_M^2 + k_{AML}^\alpha x_M^2 + k_C x_S^2 + k_{AL}^\beta y_S^2 + k_{AML}^\beta y_M^2 + k_{PIL}^\alpha x_I^2 + k_{PIL}^\beta y_I^2 \\ + k_{IMJ}^\alpha (x_M - x_I)^2 + k_{ISJ}^\alpha (x_I - x_S)^2 + k_{IMJ}^\beta (y_{MI} - x_M)^2 \\ + k_{ISJ}^\beta (y_{IS} - x_I)^2) + \frac{1}{4} k_{AL3} x_S^4\end{aligned}\quad (4.13)$$

where:

y - displacement of the structure in the Maxwell system.

Meanwhile, the Rayleigh function of dissipation (Φ) is defined as:

$$\begin{aligned}\Phi = \frac{1}{2} (c_{TM} \dot{x}_M^2 + c_{AML} (\dot{y}_M - \dot{x}_M)^2 + c_C \dot{x}_S^2 + c_{AL} (\dot{x}_S - \dot{y}_S)^2 \\ + c_{PIL} (\dot{x}_I - \dot{y}_I)^2 + c_{IMJ} (\dot{y}_{MI} - \dot{x}_I^2) + c_{ISJ} (\dot{y}_{IS} - \dot{x}_S)^2)\end{aligned}\quad (4.14)$$

where:

$\dot{y}$ - velocity of the structure in the Maxwell system.

The differential equations of motion (in Maxwell system - letter "M" in the equations) are derived by substituting Equations 4.6, 4.13, and 4.14 into Equation

(4.5), leads to:

$$\begin{aligned}
 \ddot{x}_M m_M + \tilde{k}_{11}^M x_M + \tilde{k}_{12}^M x_I + \tilde{k}_{17}^M y_{MI} + \tilde{c}_{11}^M \dot{x}_M + \tilde{c}_{14}^M \dot{y}_M &= Q_0 \cos(\omega t) \\
 \ddot{x}_I m_I + \tilde{k}_{21}^M x_M + \tilde{k}_{22}^M x_I + \tilde{k}_{23}^M x_S + \tilde{k}_{28}^M y_{IS} + \tilde{c}_{22}^M \dot{x}_I + \tilde{c}_{25}^M \dot{y}_I + \tilde{c}_{27}^M \dot{y}_{MI} &= 0 \\
 \ddot{x}_S m_S + \tilde{k}_{32}^M x_I + \tilde{k}_{33}^M x_S + \tilde{c}_{33}^M \dot{x}_S + \tilde{c}_{36}^M \dot{y}_S + \tilde{c}_{38}^M \dot{y}_{IS} + \bar{k}_{AL3} x_S^3 &= 0 \\
 \tilde{k}_{44}^M y_M + \tilde{c}_{41}^M \dot{x}_M + \tilde{c}_{44}^M \dot{y}_M &= 0 \\
 \tilde{k}_{55}^M y_I + \tilde{c}_{52}^M \dot{x}_I + \tilde{c}_{55}^M \dot{y}_I &= 0 \\
 \tilde{k}_{66}^M y_S + \tilde{c}_{63}^M \dot{x}_S + \tilde{c}_{66}^M \dot{y}_S &= 0 \\
 \tilde{k}_{71}^M x_M + \tilde{k}_{77}^M y_{MI} + \tilde{c}_{72}^M \dot{x}_I + \tilde{c}_{77}^M \dot{y}_{MI} &= 0 \\
 \tilde{k}_{82}^M x_I + \tilde{k}_{88}^M y_{IS} + \tilde{c}_{83}^M \dot{x}_S + \tilde{c}_{88}^M \dot{y}_{IS} &= 0
 \end{aligned} \tag{4.15}$$

where:

$$\begin{aligned}
 \tilde{k}_{11}^M &= k_{TM} + k_{AML}^\alpha + k_{IMJ}^\alpha + k_{IMJ}^\beta, & \tilde{k}_{12}^M &= -k_{IMJ}^\alpha, & \tilde{k}_{17}^M &= -k_{IMJ}^\beta \\
 \tilde{k}_{21}^M &= -k_{IMJ}^\alpha, & \tilde{k}_{22}^M &= k_{PIL}^\alpha + k_{IMJ}^\alpha + k_{ISJ}^\alpha + k_{ISJ}^\beta, & \tilde{k}_{23}^M &= -k_{ISJ}^\alpha, \\
 \tilde{k}_{28}^M &= -k_{ISJ}^\beta, & \tilde{k}_{32}^M &= -k_{ISJ}^\alpha, & \tilde{k}_{33}^M &= k_C + k_{ISJ}^\alpha, & \tilde{k}_{44}^M &= k_{AML}^\beta, \\
 \tilde{k}_{55}^M &= k_{PIL}^\beta, & \tilde{k}_{66}^M &= k_{AL}^\beta, & \tilde{k}_{77}^M &= k_{IMJ}^\beta, \\
 \tilde{k}_{88}^M &= k_{ISJ}^\beta, & \tilde{k}_{71}^M &= -k_{IMJ}^\beta, & \tilde{k}_{82}^M &= -k_{ISJ}^\beta, \tag{4.16} \\
 \tilde{c}_{11}^M &= c_{TM} + c_{AML}, & \tilde{c}_{14}^M &= -c_{AML}, & \tilde{c}_{22}^M &= c_{PIL} + c_{IMJ}, & \tilde{c}_{25}^M &= -c_{PIJ}, \\
 \tilde{c}_{27}^M &= -c_{IMJ}, & \tilde{c}_{33}^M &= c_C + c_{AL} + c_{ISJ}, & \tilde{c}_{36}^M &= -c_{AL}, & \tilde{c}_{38}^M &= -c_{ISJ}, \\
 \tilde{c}_{41}^M &= -c_{AML}, & \tilde{c}_{44}^M &= c_{AML}, & \tilde{c}_{52}^M &= -c_{PIL}, & \tilde{c}_{55}^M &= c_{PIL}, & \tilde{c}_{63}^M &= -c_{AL}, \\
 \tilde{c}_{66}^M &= c_{AL}, & \tilde{c}_{72}^M &= -c_{IMJ}, & \tilde{c}_{77}^M &= c_{IMJ}, & \tilde{c}_{83}^M &= -c_{ISJ}, & \tilde{c}_{88}^M &= c_{ISJ}
 \end{aligned}$$

By introducing the dimensionless time and frequency, and coordinates $y_1 - y_3$, the equations of motion are reformulated as presented in Equation 4.16. The new dimensionless coefficients and coordinates are defined as follows:

$$\begin{aligned}
 \tilde{k}_{ij} &= \tilde{k}_{ij}^M / (m_m \omega_0^2), & \tilde{c}_{ij} &= \tilde{c}_{ij}^M / (m_m \omega_0), & \gamma_3 &= k_{AL3} x_{st}^2 / (m_m \omega_0^2), \\
 i, j &= 1 \dots 8q = Q / (m_m \omega_0^2 x_{st}), & y_1 &= y_M / x_{st}, \\
 y_2 &= x_I / x_{st}, & y_3 &= x_S / x_{st}, & y_{12} &= y_{MI} / x_{st}, & y_{23} &= y_{IS} / x_{st}
 \end{aligned} \tag{4.17}$$

Final, dimensionless form of the differential equations of motion, that are used to build numerical Maxwell type model, are shown below:

$$\begin{aligned}
\ddot{x}_1 + \tilde{k}_{11}x_1 + \tilde{k}_{12}x_2 + \tilde{k}_{17}y_{12} + \tilde{c}_{11}\dot{x}_1 + \tilde{c}_{14}\dot{y}_1 &= q_0 \cos(\Omega t) \\
\ddot{x}_2 m_2 + \tilde{k}_{21}x_1 + \tilde{k}_{22}x_2 + \tilde{k}_{23}x_3 + \tilde{k}_{28}y_{23} + \tilde{c}_{22}\dot{x}_2 + \tilde{c}_{25}\dot{y}_2 + \tilde{c}_{27}\dot{y}_{12} &= 0 \\
\ddot{x}_3 m_3 + \tilde{k}_{32}x_2 + \tilde{k}_{33}x_3 + \tilde{c}_{33}\dot{x}_3 + \tilde{c}_{36}\dot{y}_3 + \tilde{c}_{38}\dot{y}_{23} + \gamma_3 x_3^3 &= 0 \\
\tilde{k}_{44}y_1 + \tilde{c}_{41}\dot{x}_1 + \tilde{c}_{44}\dot{y}_1 &= 0 \\
\tilde{k}_{55}y_2 + \tilde{c}_{52}\dot{x}_2 + \tilde{c}_{55}\dot{y}_2 &= 0 \\
\tilde{k}_{66}y_3 + \tilde{c}_{63}\dot{x}_3 + \tilde{c}_{66}\dot{y}_3 &= 0 \\
\tilde{k}_{71}x_1 + \tilde{k}_{77}y_{12} + \tilde{c}_{72}\dot{x}_2 + \tilde{c}_{77}\dot{y}_{12} &= 0 \\
\tilde{k}_{82}x_2 + \tilde{k}_{88}y_{23} + \tilde{c}_{83}\dot{x}_3 + \tilde{c}_{88}\dot{y}_{23} &= 0
\end{aligned} \tag{4.18}$$

Based on the derived equations, a dynamic model was implemented in MATLAB, Simulink to analyze the system's behavior under various conditions. The Simulink environment was chosen for its robust numerical solvers and modular approach, which facilitated the integration of nonlinearities and subsystem interactions. The simulation results, including FRFs, are presented and discussed in the subsequent chapters.

4.2 Implanted Middle Ear

In the model of the implanted middle ear, the three auditory ossicles are connected in the same way as shown in the previous section. Additionally, the FMT is attached to the second ossicle (the incus), represented by the second mass m_I . The FMT consist of the case which contains the mass driven by a force $P \cos(\omega t)$. The FMT is connected to the second mass via a rigid clip (shown in the Figure 4.3. In reallity the floating mass is excited by an electromagnetic field in this case there is simplification that changes the electrical excitation that is usually in FMT to the mechanical excitation [2].

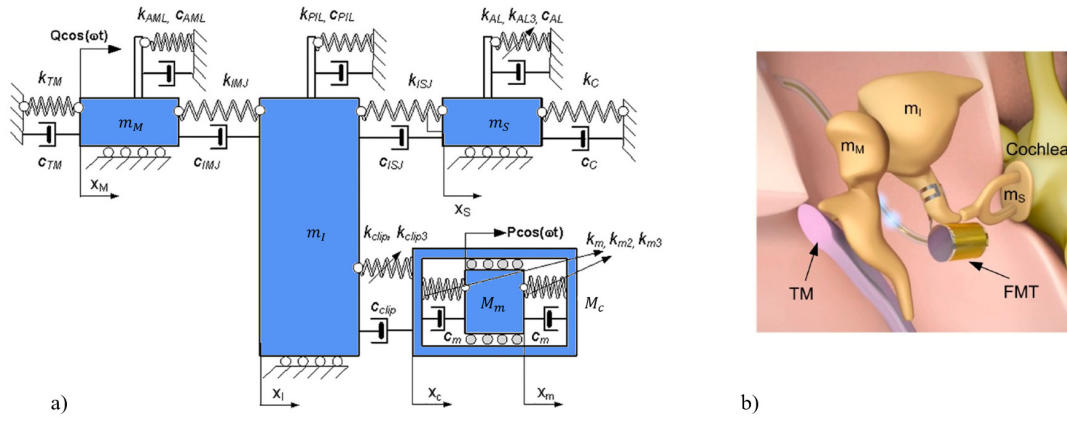


Figure 4.3. (a) The 5-DOF LPM incorporating a floating mass transducer with double excitation $Q \cos(\omega t)$ and $P \cos(\omega t)$; (b) an illustration of the implanted human middle ear [www.medel.pro].

The stiffness of the coupler is assumed to be nonlinear [19] and adaptable to individual patient requirements, which is crucial for maintenance and reliability. As a result, the governing differential dimensionless equations for the implanted human middle ear system are defined as follows:

$$\begin{aligned}
 \ddot{x}_M m_M + \tilde{k}_{11} x_M + \tilde{k}_{12} x_I + \tilde{c}_{11} \dot{x}_M + \tilde{c}_{12} \dot{x}_I &= 0 \\
 \ddot{x}_I m_I + \tilde{k}_{21} x_M + \tilde{k}_{22} x_I + \tilde{k}_{23} x_S + \tilde{k}_{24} x_c + \tilde{\gamma}_{24} (x_I - x_c)^3 + \\
 \tilde{c}_{21} \dot{x}_M + \tilde{c}_{22} \dot{x}_I + \tilde{c}_{23} \dot{x}_S + \tilde{c}_{24} \dot{x}_c &= 0 \\
 \ddot{x}_S m_S + \tilde{k}_{32} x_I + \tilde{k}_{33} x_S + \tilde{c}_{32} \dot{x}_I + \tilde{c}_{33} \dot{x}_S + \tilde{\gamma}_3 x_S^3 &= 0 \\
 \ddot{x}_c m_c + \tilde{k}_{42} x_I + \tilde{k}_{44} x_c + \tilde{k}_{45} x_m + \tilde{c}_{42} \dot{x}_I + \tilde{c}_{44} \dot{x}_c + \tilde{c}_{45} \dot{x}_m - \\
 \tilde{\gamma}_{24} (x_I - x_c)^3 - \tilde{\beta}_{45} (x_c - x_m)^2 + \tilde{\gamma}_{45} (x_c - x_m)^3 &= 0 \\
 \ddot{x}_m m_m + \tilde{k}_{54} x_c + \tilde{k}_{55} x_m + \tilde{c}_{54} \dot{x}_c + \tilde{c}_{55} \dot{x}_m + \tilde{\beta}_{45} (x_c - x_m)^2 - \\
 \tilde{\gamma}_{45} (x_c - x_m)^3 &= P \cos(\omega t)
 \end{aligned} \tag{4.19}$$

where:

$$\begin{aligned}
 \tilde{k}_{11} &= k_{TM} + k_{AML} + k_{IMJ}, \tilde{k}_{12} = -k_{IMJ}, \tilde{c}_{11} = c_{TM} + c_{AML} + c_{IMJ}, \\
 \tilde{c}_{12} &= -c_{IMJ}, \tilde{k}_{21} = k_{12}, \tilde{k}_{22} = k_{PIL} + k_{ISJ} + k_{IMJ} + k_{clip}, \tilde{k}_{23} = -k_{ISJ}, \\
 \tilde{k}_{24} &= -k_{clip}, \tilde{\gamma}_{24} = k_{clip3}, \tilde{c}_{21} = -c_{IMJ}, \tilde{c}_{22} = c_{PIL} + c_{ISJ} + c_{IMJ} + c_{clip}, \\
 \tilde{c}_{23} &= -c_{ISJ}, \tilde{c}_{24} = -c_{clip}, \tilde{\gamma}_{24} = k_{clip3}, \tilde{k}_{32} = k_{23}, \\
 \tilde{k}_{33} &= k_{AL} + k_{ISJ} + k_C, \tilde{c}_{32} = c_{23}, \tilde{c}_{33} = c_{AL} + c_{ISJ} + c_C, \tilde{\gamma}_3 = k_{AL3}, \\
 \tilde{k}_{42} &= k_{24}, \tilde{k}_{44} = k_{clip} + k_m, \tilde{k}_{45} = -k_m, \tilde{c}_{42} = c_{24}, \tilde{c}_{44} = c_{clip} + c_m, \\
 \tilde{c}_{45} &= -c_m, \tilde{\gamma}_{45} = k_{m3}, \tilde{\beta}_{45} = k_{m2}, \tilde{k}_{54} = k_{45}, \\
 \tilde{k}_{55} &= k_m = -k_{54}, \tilde{c}_{54} = c_{45}, \tilde{c}_{55} = c_m = -c_{54}.
 \end{aligned} \tag{4.20}$$

To streamline the subsequent analysis, the dimensionless time, frequency and coordinates $x_4 - x_5$ (the coordinates $(x_1 - x_3)$ where shown in previous section) are defined using the following expressions:

$$x_4 = x_c/x_{st}, \quad x_5 = x_m/x_{st} \tag{4.21}$$

The dimensionless equations of motion are finally expressed as:

$$\begin{aligned}
 \ddot{x}_1 + k_{11}x_1 + k_{12}x_2 + c_{11}\dot{x}_1 + c_{12}\dot{x}_2 &= 0 \\
 \ddot{x}_2m_2 + k_{21}x_1 + k_{22}x_2 + k_{23}x_3 + k_{24}x_4 + c_{21}\dot{x}_1 + c_{22}\dot{x}_2 + \\
 c_{23}\dot{x}_3 + c_{24}\dot{x}_4 + \gamma_{24}(x_2 - x_4)^3 &= 0 \\
 \ddot{x}_3m_3 + k_{32}x_2 + k_{33}x_3 + c_{32}\dot{x}_2 + c_{33}\dot{x}_3 + \gamma_3x_3^3 &= 0 \\
 \ddot{x}_4m_4 + k_{42}x_2 + k_{44}x_4 + k_{45}x_5 + c_{42}\dot{x}_2 + c_{44}\dot{x}_4 + c_{45}\dot{x}_5 \\
 -\gamma_{24}(x_2 - x_4)^3 - \beta_{45}(x_4 - x_5)^2 + \gamma_{45}(x_4 - x_5)^3 &= 0 \\
 \ddot{x}_5m_5 + k_{54}x_4 + k_{55}x_5 + c_{54}\dot{x}_4 + c_{55}\dot{x}_5 + \\
 \beta_{45}(x_4 - x_5)^2 - \gamma_{45}(x_4 - x_5)^3 &= p \cos(\Omega\tau)
 \end{aligned} \tag{4.22}$$

where the dimensionless parameters are given by:

$$\begin{aligned}
k_{11} &= \tilde{k}_{11}/(m_M \omega_0^2), k_{12} = \tilde{k}_{12}/(m_M \omega_0^2), c_{11} = \tilde{c}_{11}/(m_M \omega_0), \\
c_{12} &= \tilde{c}_{12}/(m_M \omega_0), k_{21} = \tilde{k}_{21}/(m_M \omega_0^2), k_{22} = \tilde{k}_{22}/(m_M \omega_0^2), \\
k_{23} &= \tilde{k}_{23}/(m_M \omega_0^2), k_{24} = \tilde{k}_{24}/(m_M \omega_0^2), c_{21} = \tilde{c}_{21}/(m_M \omega_0), \\
c_{22} &= \tilde{c}_{22}/(m_M \omega_0), c_{23} = \tilde{c}_{23}/(m_M \omega_0), c_{24} = \tilde{c}_{24}/(m_M \omega_0), \\
\gamma_{24} &= \tilde{\gamma}_{24} x_{st}^2 / (m_M \omega_0^2), \\
k_{32} &= \tilde{k}_{32}/(m_M \omega_0^2), k_{33} = \tilde{k}_{33}/(m_M \omega_0^2), c_{32} = \tilde{c}_{32}/(m_M \omega_0), \\
c_{33} &= \tilde{c}_{33}/(m_M \omega_0), \gamma_3 = \tilde{\gamma}_3 x_{st}^2 / (m_M \omega_0^2), \quad (4.23) \\
k_{42} &= \tilde{k}_{42}/(m_M \omega_0^2), k_{44} = \tilde{k}_{44}/(m_M \omega_0^2), k_{45} = \tilde{k}_{45}/(m_M \omega_0^2), \\
c_{42} &= \tilde{c}_{42}/(m_M \omega_0), c_{44} = \tilde{c}_{44}/(m_M \omega_0), c_{45} = \tilde{c}_{45}/(m_M \omega_0), \\
\gamma_{45} &= \tilde{\gamma}_{45} x_{st}^2 / (m_M \omega_0^2), \beta_{45} = \tilde{\beta}_{45} x_{st} / (m_M \omega_0^2), \\
k_{54} &= \tilde{k}_{54}/(m_M \omega_0^2), k_{55} = \tilde{k}_{55}/(m_M \omega_0^2), \\
c_{54} &= \tilde{c}_{54}/(m_M \omega_0), c_{55} = \tilde{c}_{55}/(m_M \omega_0), \\
\omega &= \Omega \omega_0, p = P / (m_M x_{st} \omega_0^2).
\end{aligned}$$

In a similar way to the previous section using the derived equations, a dynamic model was developed in MATLAB/Simulink to evaluate the system's behavior under varying conditions. The resulting simulation data, including the FRFs, is presented and analyzed in later chapters.

4.3 Double Stimulated IME

When an implant is placed on the long or short process of the incus, the tympanic membrane is still transferring the sound from outer to the middle ear, regardless of its condition. Sound can still pass through the membrane to reach the middle ear ossicles, causing movement and resulting in double stimulation. Additionally, as an electronic system, the implant introduces a delay due to factors like signal processing, communication, and computation, which contribute to overall latency. In this case the phenomenon of double excitation in IMEs is investigated using the 5-DOF lumped parameter model (shown in the previous section). The double excitation issue arises because, even with an implant attached to the incus, sound waves continue to stimulate the eardrum, setting the ossicles in motion. Consequently, the

ossicles are activated by both the MEI and the sound reaching the tympanic membrane. This represents a novel and critical challenge that has not been thoroughly explored in existing literature.

In this case, the model is the same as the one from the previous section, but it includes an additional excitation applied to the first mass (shown in the Figure 4.3) (malleus),- m_M . Furthermore, to analyze the system's delay, a Heaviside step function is introduced, allowing for the delayed application of excitation to the implant. With this in mind, the equations governing the system are as follows:

$$\begin{aligned}
 \ddot{x}_M m_M + \tilde{k}_{11} x_M + \tilde{k}_{12} x_I + \tilde{c}_{11} \dot{x}_M + \tilde{c}_{12} \dot{x}_I &= Q \cos(\omega t) \\
 \ddot{x}_I m_I + \tilde{k}_{21} x_M + \tilde{k}_{22} x_I + \tilde{k}_{23} x_S + \tilde{k}_{24} x_c + \tilde{\gamma}_{24} (x_I - x_c)^3 + \\
 \tilde{c}_{21} \dot{x}_M + \tilde{c}_{22} \dot{x}_I + \tilde{c}_{23} \dot{x}_S + \tilde{c}_{24} \dot{x}_c &= 0 \\
 \ddot{x}_S m_S + \tilde{k}_{32} x_I + \tilde{k}_{33} x_S + \tilde{c}_{32} \dot{x}_I + \tilde{c}_{33} \dot{x}_S + \tilde{\gamma}_3 x_S^3 &= 0 \\
 \ddot{x}_c m_c + \tilde{k}_{42} x_I + \tilde{k}_{44} x_c + \tilde{k}_{45} x_m + \tilde{c}_{42} \dot{x}_I + \tilde{c}_{44} \dot{x}_c + \tilde{c}_{45} \dot{x}_m - \\
 \tilde{\gamma}_{24} (x_I - x_c)^3 - \tilde{\beta}_{45} (x_c - x_m)^2 + \tilde{\gamma}_{45} (x_c - x_m)^3 &= 0 \\
 \ddot{x}_m m_m + \tilde{k}_{54} x_c + \tilde{k}_{55} x_m + \tilde{c}_{54} \dot{x}_c + \tilde{c}_{55} \dot{x}_m + \tilde{\beta}_{45} (x_c - x_m)^2 - \\
 \tilde{\gamma}_{45} (x_c - x_m)^3 &= H(\tau_1) \cdot P \cos(\omega t)
 \end{aligned} \tag{4.24}$$

Then, Heaviside step function $H(\tau_1)$ is defined as follows:

$$H(\tau_1) = \begin{cases} 0 & \text{for } t < \tau_1 \\ 1 & \text{for } t \geq \tau_1 \end{cases} \tag{4.25}$$

The dimensionless equations of motion, with the modifications discussed earlier, are expressed as presented in the previous section.

Experimental Research

The objective of this study is to validate a comprehensive model representing the biomechanical behavior of a intact human ear and to establish a set of standard reference characteristics. These reference characteristics will serve as a baseline for evaluating and comparing the functional performance of both the natural ear model and hearing implants.

To achieve this objective, a series of experiments is conducted using human temporal bone specimens, following a defined in the literature experimental protocol outlined in the subsequent sections. This methodological approach ensures consistency and reliability in data collection.

Central to the study is the measurement and analysis of the FRF of the intact middle ear which provides a quantitative representation of how the middle ear transmits sound energy across various frequencies, making it a critical parameter for understanding both normal ear mechanics as well as with implant. The obtained data will contribute to a clearer understanding of the ear's acoustic transmission properties and facilitate comparisons between the natural ear and IMEHD.

5.1 Methodology

To investigate the frequency response functions of the the stapes footplate, a series of experiments were performed using five human temporal bone specimens. Standard otolaryngological procedures were employed for specimen preparation, which

included the removal of soft tissue, followed by an antromastoidectomy with posterior tympanotomy. The mastoid segment of the facial nerve was excised to allow direct access to the stapes arch and footplate.

A cylindrical tube measuring 9 mm in diameter and 27 mm in length, designed to act as the artificial external ear canal, was securely affixed to each specimen using epoxy resin. This tube featured two ports: one for an ear microphone (ER-7C, Etymotic Research) and another for a sound source (speaker ER2, Etymotic Research), as illustrated in Figure 5.1. The preparation procedure is consistent to established standards and aligns with methodologies described in different studies [79, 78, 108].

Reflective tape segments, each weighing less than 0.05 mg, were carefully placed on the SF to enhance the accuracy of measurements using LDV. Each specimen was embedded in dental cement and securely mounted in a temporal bone block holder (Karl Storz), which was put on an anti-vibration table. The entire setup was housed in a soundproof chamber to eliminate external noise interference.

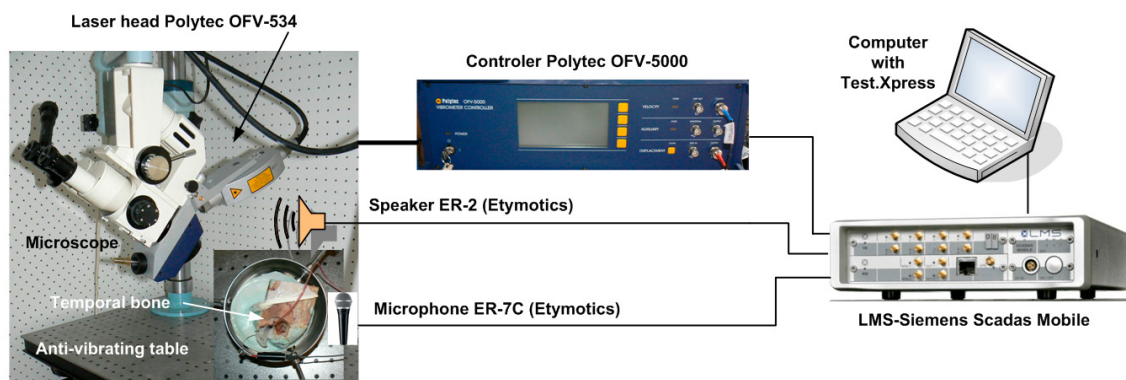


Figure 5.1. *Experimental setup for the human middle ear measurements.*

A chirp signal spanning frequencies from 0.25 to 8 kHz was introduced to the outer ear artificial canal, encompassing the typical range of human hearing. The system's response, specifically the velocities of designated reference points on the SF, was recorded using an LDV system (Polytec GmbH, Waldbronn, Germany), which consist of an OFV-5000 controller and an OFV-534 laser scanning head. The He-Ne laser beam was directed at the retro-reflective markers through the artificial ear canal. Velocity signals captured by the LDV system were transmitted

to the LMS-Siemens SCADAS Mobile data acquisition analyzer and subsequently processed using TestXpress software on a PC (refer to Figure 5.1).

The experiment was conducted in the Department of Applied Mechanics as part of the NCN project (No. 2014/13/B/ST8/04047) with approval from the ethics committee of the Medical University of Lublin (No. KE-0254/298/2014). Results are typically presented as transfer functions that correlate vibrations or velocities of the stapes (or other middle ear structures) to pressure stimuli within the ear canal. In this study, the velocity as well as displacement of the SF was specifically measured.

5.2 Experimental Results

To analyze the vibration of the intact and implanted human middle ear, a classical transfer function is engaged. The stapes velocity and displacement, measured using LDV, serves as the output, while the sound pressure, captured by a microphone, acts as the input. Listed below cases will be examined:

1. H_{TV} function that represents the ratio of stapes velocity to sound pressure in IOC.
2. H_{TV}^{Aided} function that represents the ratio of stapes velocity to sound pressure where the implant is attached and turned on.
3. $H_{TV}^{Unaided}$ function that represents the ratio of stapes velocity to sound pressure where the implant is attached and turned off.
4. H_{TD} function that corresponds to the ratio of stapes displacement to sound pressure.

Both mentioned functions are called as Frequency Response Function. Experimental measurements focused on the human ear was conducted in four distinct configurations, enabling a detailed analysis of its functioning under various conditions. Below is a description of each case:

1. Intact Ossicle Chain

In the first scenario, measurements were performed on a unmodified ear -

IOC. The experiment involved measuring the velocity of the stapes' vibrations in response to sound excitation while simultaneously recording the generated acoustic pressure. Those results are presented in Figure 5.2.

2. IME – sound-induced excitation

The second scenario examined an ear equipped with an implant, where the vibrations of the stapes were induced solely by sound directed at the tympanic membrane, without the implant being actively utilized. The implant was attached to the long process of the incus, allowing for an analysis of its impact on the vibration characteristics under these conditions. Those results are presented in Figure 5.3.

3. IME – implant-induced excitation

In the third case, the focus was on the implant as the source of vibrations. The vibrations of the stapes were induced exclusively by the implant, and the measurements concentrated on recording the surrounding pressure. Those results are presented in Figure 5.4.

4. IME – double excitation

The final scenario involved a study where the vibrations of the stapes were induced by both sound excitation acting on the tympanic membrane and the implant. This configuration allowed for an analysis of the interaction between the two excitation methods and their combined effect on the stapes vibration characteristics. Those results are presented in Figure 5.5.

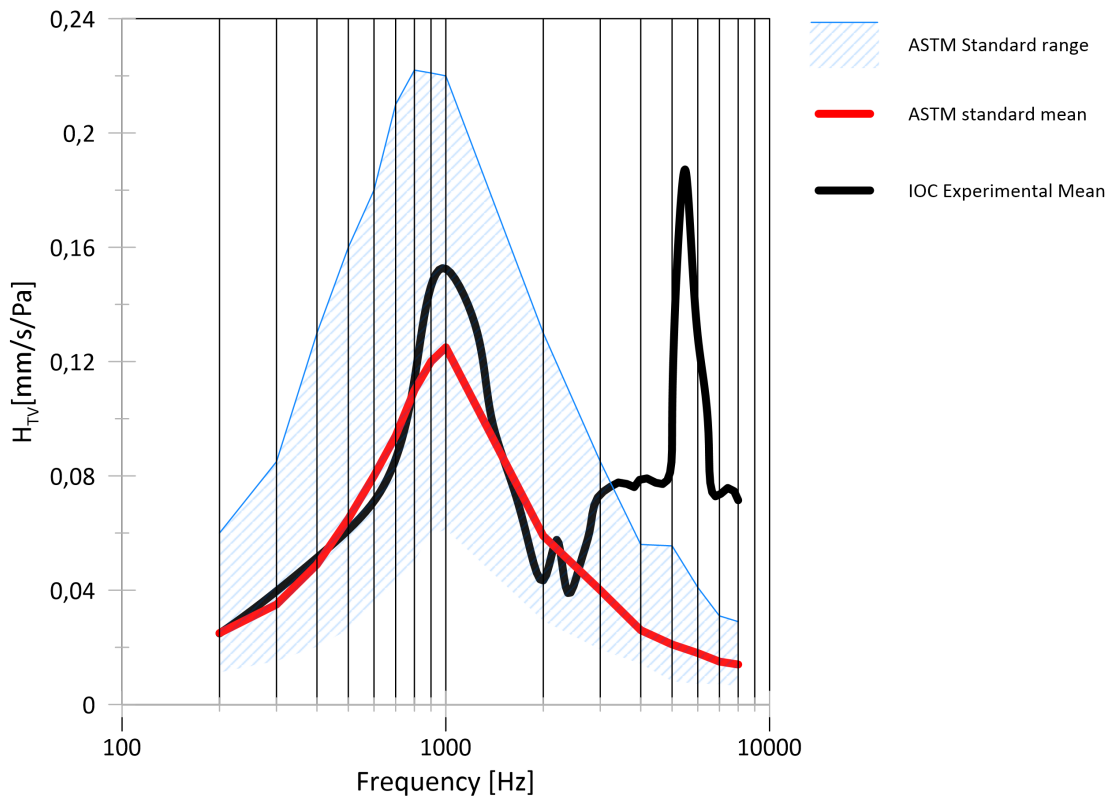


Figure 5.2. Comparison of the experimental and literature Intact HME measurements described by the transfer function H_{TV} .

As mentioned at the very beginning the measurements of the five temporal bones were performed. Figure 5.2 presents a series of experimental tests conducted on the IOC, where sound waves reaching the eardrum serve as the excitation source. The corresponding response is measured as the velocity of the stapes footplate. The figure includes a slashed area representing the range specified by the ASTM F2504 standard. This standard defines the acceptable performance range for the tested bone structure, covering frequencies between 250 Hz and 4000 Hz, however most of the references extended this region to 8000 Hz. The red curve illustrates the average response according to the ASTM standard, providing a reference for comparison. The graph displays mean of five experimental measurements performed during this study, alongside results from the literature, including the ASTM standard. In the literature [74, 3] a, a single resonance peak is consistently observed around 1 kHz which is the same as in presented experimental measurements. Additionally, a key difference between the ASTM F2504 standard [3] and literature data [74] and our measurements is the presence of a second res-

onance peak. While the ASTM F2504 standard and literature [74] do not indicate a second resonance peak for IOC, in case of those measurements there is peak at 4500 Hz. This second resonance peak aligns with results from numerical calculations presented in subsequent sections of this work. It is also consistent with findings reported by Shin et al. (2020), further validating the observation. The slight discrepancies observed in the low-frequency range across measurements may be attributed to differences in the mass of the reflective tape used in this study compared to other studies. The added mass could introduce minor variations in the vibration dynamics of the stapes footplate.

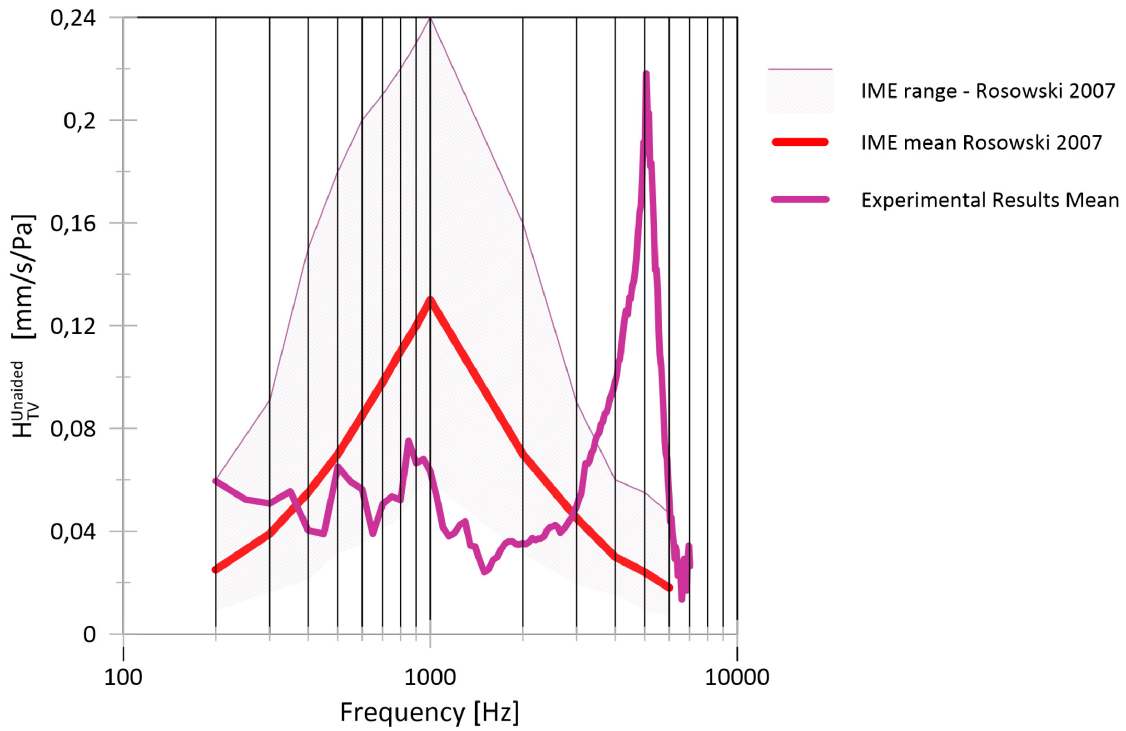


Figure 5.3. Comparison of the experimental and literature IME measurements described by the transfer function $H_{TV}^{Unaided}$.

Next the measurements of the IME for five temporal bones were conducted, while the FMT were turned off and the excitation came from the tympanic membrane. In this configuration, the excitation source is the sound reaching the eardrum, while the response is measured as the velocity of the stapes base. Here, the implant acts as an additional mass within the system, rather than as a source of excitation. The graph (see Figure 5.3) incorporates a shaded area that represents the range of

correctness established in [74]. Additionally, a red curve shows the average results, serving as a reference for comparison with the experimental data. Average of the five measurements is shown in the figure. The curve that represents average lies within the range for 200 Hz to 1 kHz and from 1800 Hz to 3500 Hz, while it falls below between 1000 Hz and 1800 Hz. First resonance peak occurs earlier in comparison to the ASTM F2504 at 800 Hz. However, the second resonance peak remains visible and occurs at a frequency of approximately 5000 Hz, similar to the results seen in Figure 5.2. The addition of the implant as an extra mass in the system appears to significantly alter the dynamic behavior of the ossicular chain.

Following the order mentioned at the beginning of the chapter the experimental studies were conducted on an IME, where the excitation source is a FMT. The response is measured as the $H_{TV}^{Unaided}$. In this configuration, the velocity measurements can be referenced to either the voltage driving the implant (in volts) or the pressure level. While referencing the voltage is more precise, as it directly correlates to the excitation force applied to the implant, pressure measurements are included to provide a basis for comparison with numerical simulations in later sections. These simulations use mechanical force as the excitation source, making pressure a more compatible reference for aligning experimental and computational results. The figure 5.4 includes a hatched area that defines the validity range established in [74]. While The red line represents the average results, serving as a benchmark for evaluating the experimental data.

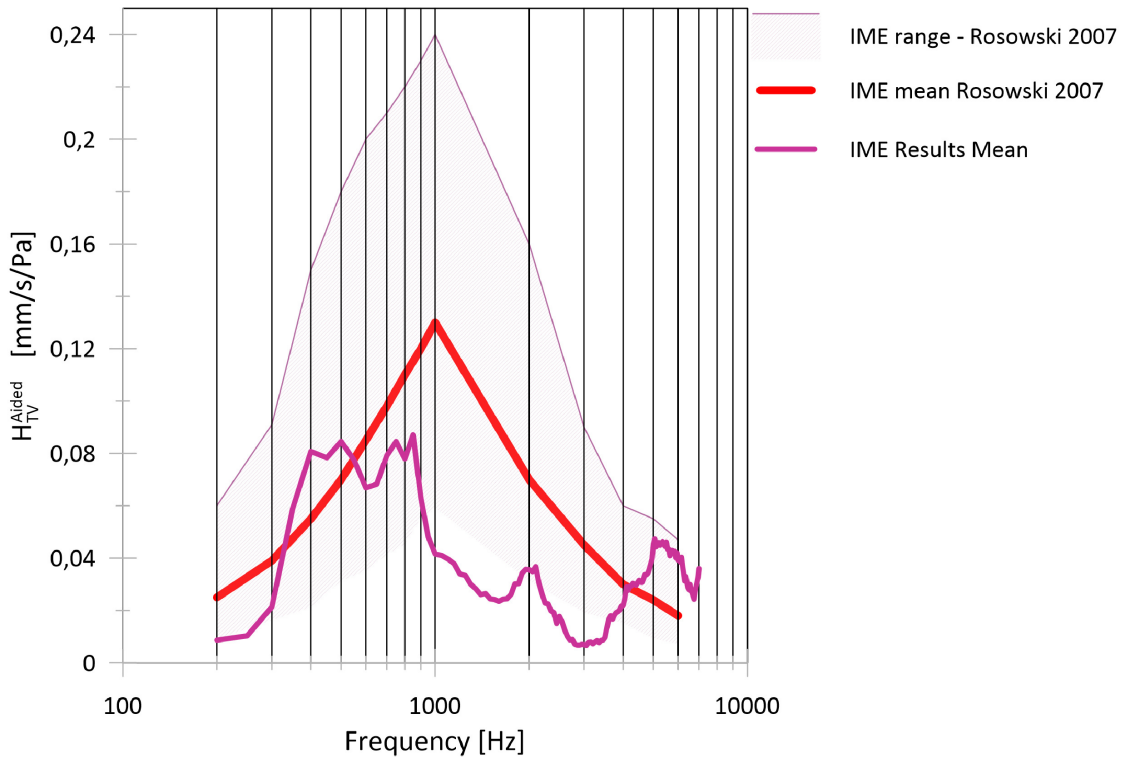


Figure 5.4. Comparison of the experimental and literature IME measurements described by the transfer function H_{TV}^{Aided} .

Mean of the five measurements is shown in the Figure 5.4 where response is measured as the H_{TV}^{Aided} . For the experimental measurement, the first resonance peak is shifted downward from its typical location at 1000 Hz to approximately 750 Hz. The second resonance peak is observed at 5000 Hz, lying in the range specified in the literature. However the mean curve falls below the literature range between 900 and 2000 Hz and between 2100 and 3500 Hz. These observed differences in resonance behavior and curve deviations may be attributed to several factors. Variability in the experimental results could arise from referencing the response to pressure level rather than the more precise voltage measurement. Additionally, the voltage applied to the implant during the experiments may have been too low to produce a response fully comparable to the standard reference data.

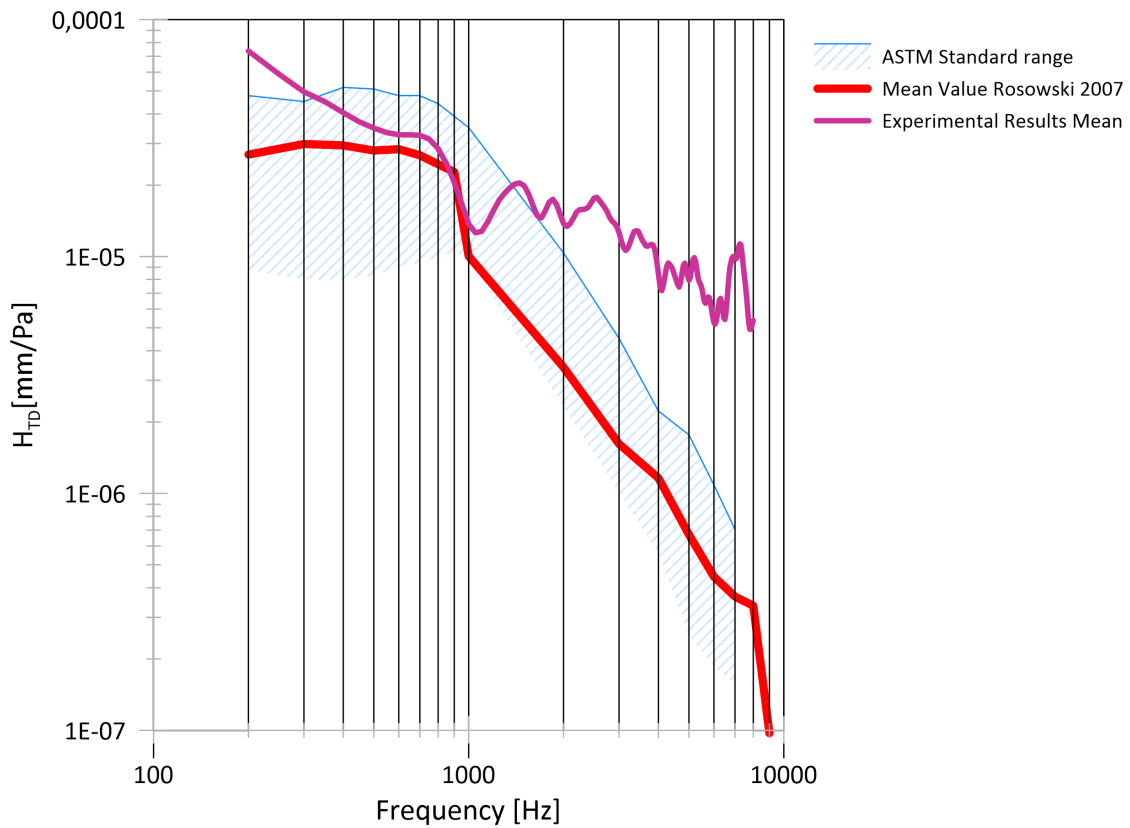


Figure 5.5. Comparison of the experimental and literature IME measurements described by the transfer function H_{TD} .

At the end, according to the research plan, the experimental results for an ear with an implant, subjected to double excitation sources: sound applied the eardrum and a FMT were conducted. The response is measured as the displacement of the stapes footplate, offering detailed insights into the dynamic behavior of the implanted middle ear system under complex excitation conditions. In this configuration, the pressure used to relate the displacement combines the pressure at the eardrum and the pressure level in the implant system. This combined pressure reflects the total driving force acting on the ossicular chain. To accommodate the frequent cyclical fluctuations observed in the data and improve clarity, the Y-axis is presented on a logarithmic scale. A red line as in previous figures indicates the average results from Rosowski's work, providing a reference point for evaluating the measurements. Mean of the five measurements is shown in the Figure 5.5. The mean curve is within the range of ASTM F2504 between 300 and 1600 Hz. Above

1600 Hz, the displacement curves fall outside the range specified in the literature. This trend suggests significant deviations in the dynamic behavior of the implanted middle ear when subjected to double excitation. The pronounced fluctuations and deviations observed in the measurements are likely influenced by phase-shifted forces acting on the ossicular chain. The interaction between the sound-driven excitation at the eardrum and the mechanical excitation from the FMT introduces a complex interplay of forces. These forces, when out of phase, can lead to constructive and destructive interference, which may account for the observed behavior. To better understand this phenomenon, a detailed numerical analysis of double excitation in an implanted ear is provided in a next chapter. This analysis aims to elucidate the underlying mechanics and explore how phase relationships between the excitation sources affect the system's response. Overall, Figure 4.5 highlights the challenges of interpreting dynamic behavior in a middle ear system under combined excitation. The results emphasize the need for precise control and analysis of excitation conditions, particularly in systems involving middle ear implants.

Intact ME Analysis

Based on the mathematical models detailed in the Chapter 3, numerical models are constructed within the MATLAB Simulink environment. Subsequently, numerical computations are conducted using the 4th order Runge-Kutta method (ode55) with a variable step size and a relative tolerance of 10^{-10} . In these numerical experiments, the model parameters provided in Table 6.1 are employed. The stiffness (k), damping (c), and mass (m) parameters are sourced from existing literature. The remaining parameters, are determined through an iterative process, aligning the numerical results presented as FRF with experimental data. This alignment primarily involves comparing the frequencies and amplitudes of resonances, especially the most important for hearing process - the first one.

Table 6.1. *Parameters of the middle ear used in numerical simulations, taken from experimental validation.*

Stiffness (All models) (N/m)	Damping Maxwell Ns/m	Damping Kelvin-Voigt Ns/m	Other
$k_{TM} = 300$	$c_{TM} = 0.378$	$c_{TM} = 0.06$	$m_M = 2.5e - 5kg$
$k_{AML} = 800$	$c_{AML} = 0.0004$	$c_{AML} = 0.275$	$m_I = 2.8e - 5kg$
$k_{IMJ} = 100000$	$c_{IMJ} = 0.359$	$c_{IMJ} = 0.359$	$m_S = 1.78e - 6kg$
$k_{PIL} = 400$	$c_{PIL} = 0.055$	$c_{PIL} = 0.055$	$Q_0 = 1.2e - 4$ N
$k_{ISJ} = 1350$	$c_{ISJ} = 0.0079$	$c_{ISJ} = 0.079$	$\alpha = 0.96$
$k_{AL} = 623$	$c_{AL} = 4$	$c_{AL} = 0.002$	$t_1 = 0.0884$
$k_C = 200$	$c_C = 0.011$	$c_C = 0.0017$	
$k_{AL3} = 1.3e11$			

At the outset, in Maxwell model, the stiffness ratio α is set to 0.96, as some researchers suggest that ligaments and tendons should be regarded as a composite of collagen fibers and a surrounding proteoglycan-rich matrix. Consequently, the mechanical behavior of these tissues is represented using a Maxwell-type viscoelastic material, where the elastic moduli of the matrix and fibers are $E_m=130.4$ MPa and $E_f=3500$ MPa, respectively. The total modulus is given by $E = E_m + E_f$. The coefficient α (as defined by the equations in Chapter 4) describes the relationship between the total modulus and the matrix Young's modulus as $\alpha = E_m/E$.

6.1 Frequency Response Function

The FRFs of the middle ear, also referred to as the transfer function, is widely used by researchers in both engineering and medical fields. The FRF provides a mathematical representation of the relationship between a system's input and output. In the context of the intact ear, the input is typically the pressure applied to the ear, while the output is the vibration velocity of the stapes resulting from that pressure. In this study, the experimentally obtained FRF is compared with numerical results derived from three models described in Chapter 4. The analysis emphasizes the stapes vibration due to its critical role in transmitting sound to the inner ear.

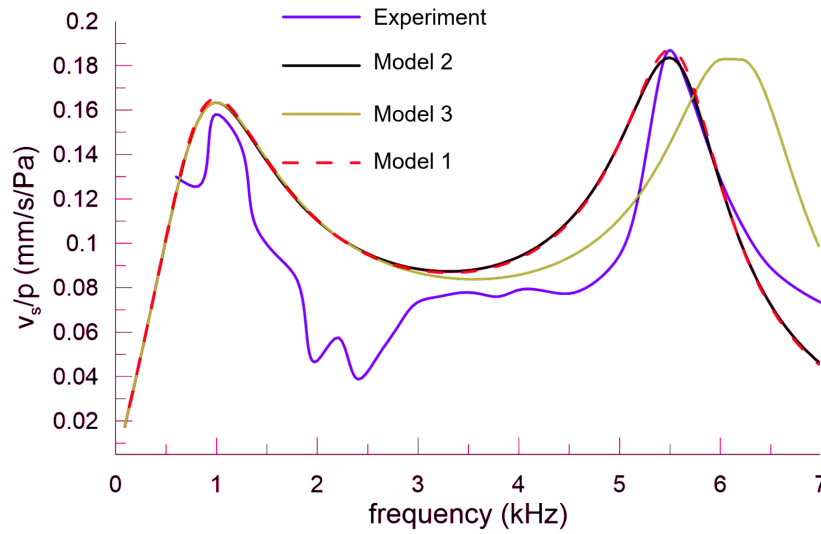


Figure 6.1. The transfer function of the middle ear, derived from the experiment (blue line) and numerical simulations (Maxwell type of viscoelasticity (Model 1) and Kelvin-Voight type of viscoelasticity (Model 2) and additionally with relaxation effect (Model 3)), is presented as the ratio of stapes velocity to sound pressure (v_s/p) plotted against the excitation frequency.

The stapes vibration measured in the experiment, shown in Figure 6.1, is related to the pressure to make comparison to the numerical results possible. The average stapes velocity from the one of the experiments is represented by the blue line in Figure 6.1. Additionally, the numerical simulation results (black, green, and red lines) are presented for comparison and verification of the three models (Maxwell type of viscoelasticity (Model 1) and Kelvin-Voight type of viscoelasticity (Model 2) and additionally with relaxation effect (Model 3)) against the experimental data. The experimental results indicate that the first resonance of the intact middle ear occurs around 1 kHz, and the second resonance is approximately 5.5 kHz. Similar resonances are anticipated for the models analyzed. Consequently, the parameters for the Maxwell model (Model 1) and the Kelvin-Voigt models (Models 2 and 3), as presented in Table 7.1, differ primarily in their damping coefficients. These damping coefficients are adjusted to match the velocity amplitude observed in the experiment. The experimental and numerical resonance frequencies for Models 1 and 2 are in agreement, which is expected given the small value of β used in the calculations. However, for Model 3, a shift in the second resonance frequency is observed. This shift occurs despite identical stiffness coefficients and is attributed to the relaxation effect. It is important to note that pure

harmonic stimulation induces harmonic vibrations of the ossicles in the intact human ear. However, this behavior may change if model parameters or excitation amplitude are altered. Therefore, the stapes vibration issue is analyzed with particular attention in the following part.

6.2 Effect of Excitation Amplitude

The excitation amplitude (pressure in the experiment and force Q in numerical analysis) is a crucial factor affecting stapes dynamics. It is well-established that a higher excitation amplitude results in stronger stapes vibration, as illustrated in Figure 6.2. However, excessive excitation can lead to irregular motion, particularly in nonlinear systems. In the current analysis, the nonlinearity of the annular ligament (k_{AL3}) induces unexpected stapes behavior, as depicted in Figure 6.3. The vertical dashed line indicates the region of periodic but not harmonic oscillations. These vibration comprise two components with the same frequency but different amplitudes, a phenomenon termed disturbed harmonic response. The blue line marks the onset of the initial disturbance in harmonic response, while the green line indicates higher-order disturbances. This means the output signal (x_3) exhibits additional harmonics. Notably, DHR occurs earlier in Model 1 compared to Models 2 and 3. The motion alteration is confined to the stapes, as the malleus and incus continue to vibrate harmonically throughout (see Fig. 6.4 where this behaviour is presented).

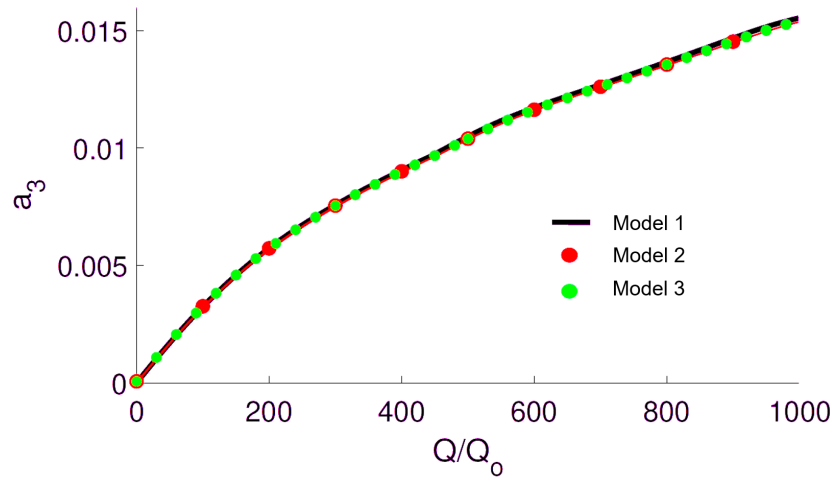


Figure 6.2. Impact of external excitation amplitude (Q) on the dimensionless stapes amplitude (a_3) near the first resonance.

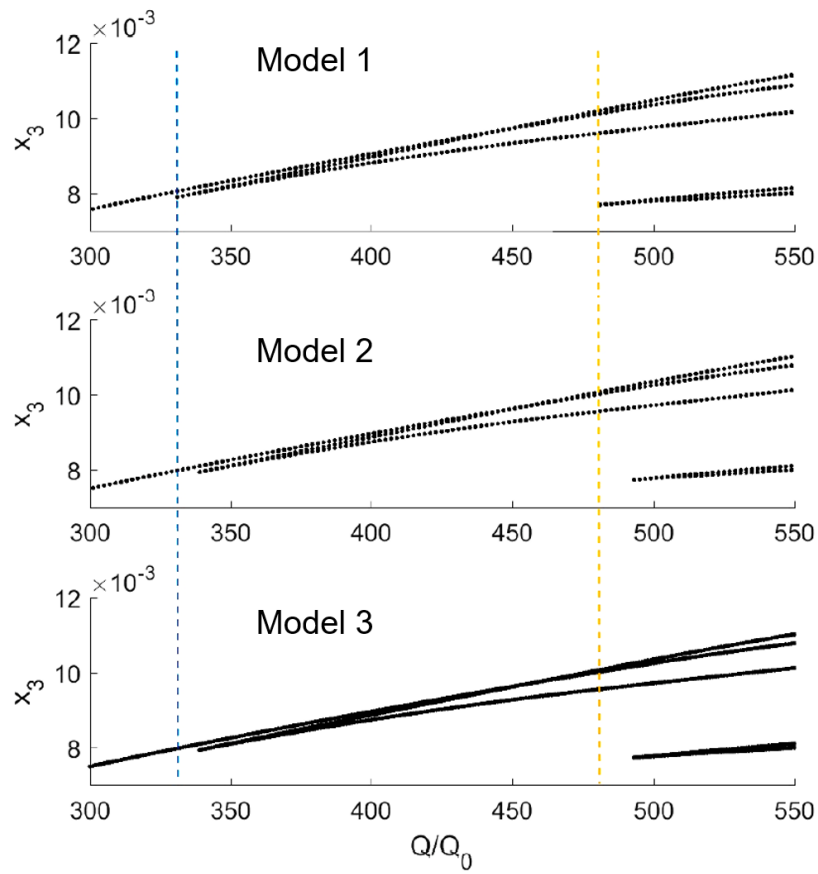


Figure 6.3. Effect of external excitation amplitude (Q) on stapes motion (x_3) near the first resonance.

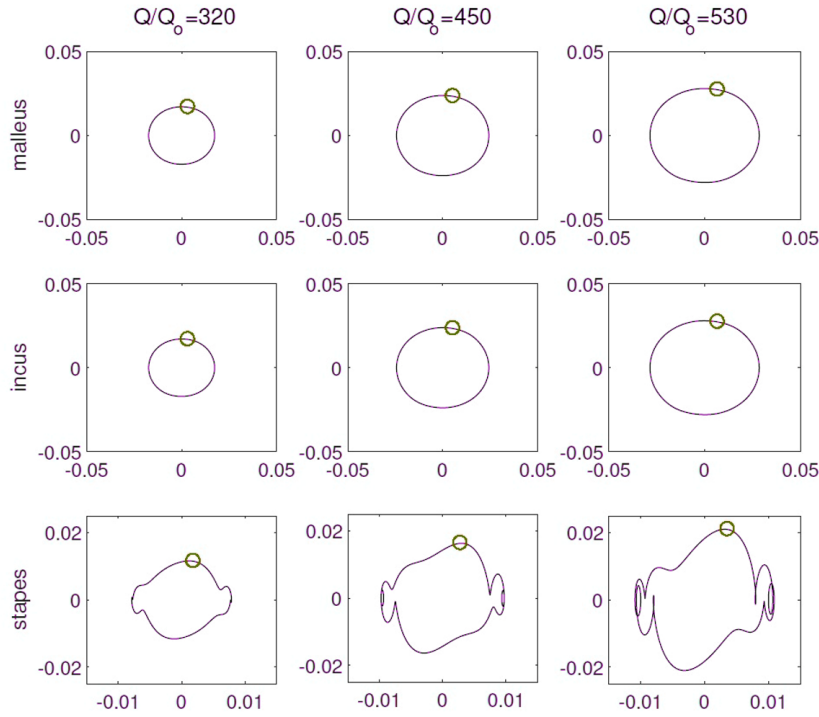


Figure 6.4. Ossicle motion depicted as phase diagrams (black) with Poincaré points (red) for Model 1 under varying external excitation amplitudes (Q/Q_0).

Figure 6.4 presents the phase diagram exclusively for Model 1, as Models 2 and 3 exhibit identical behavior. While the stapes motion varies with the excitation amplitude, the vibration period remains constant at $1T$. This indicates that the response vibrations have a period equal to 1 relative to the excitation frequency, meaning the response period matches the excitation period precisely.

6.3 Influence of Stiffness Ratio in Maxwell Model

In the FRF shown in Figure 6.1 $\alpha = 0.96$ (Model 1). The influence of α (also β as defined in equation 4.4) on the dimensionless amplitude of stapes vibration (a_3) is presented in Figure 6.5. An increase of the α (the additional spring in Maxwell viscoelastic element) causes a nonlinear increase in the vibrations amplitude. This is especially visible at higher values of the external excitation ($5Q$ and $10Q$). Thus, the stiffness ratio does not change the type of motion, but has an effect on the stapes vibration amplitude.

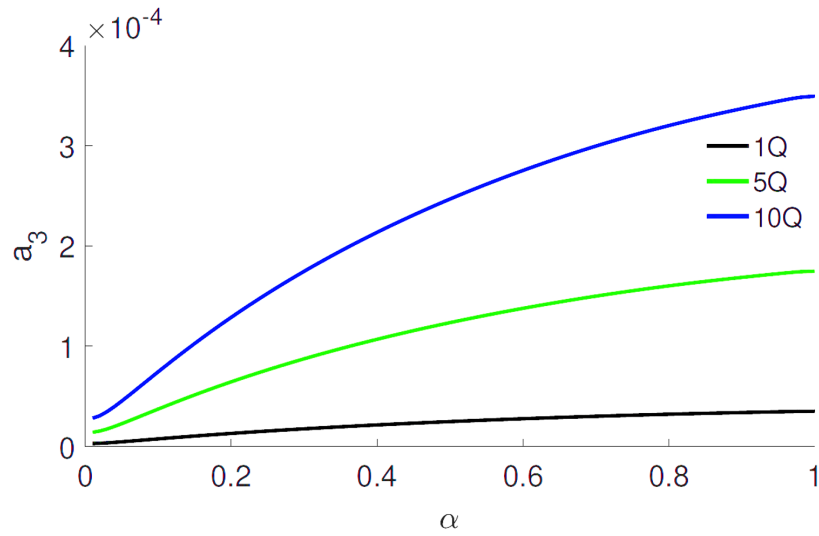


Figure 6.5. Effect of the stiffness ratio (α) in Model 1 on the dimensionless stapes amplitude (a_3).

6.4 Influence of the Relaxation Time

In dimensionless Model 3, the influence relaxation time (τ_1) on stapes vibrations is analyzed. The relaxation effect causes a shift in the second resonance, as seen in Figure 6.1. This shift in the second resonance frequency ($\Omega_2 = 6$) increases with τ_1 , as illustrated in Figure 6.6, while the vibration amplitude (a_3) decreases.

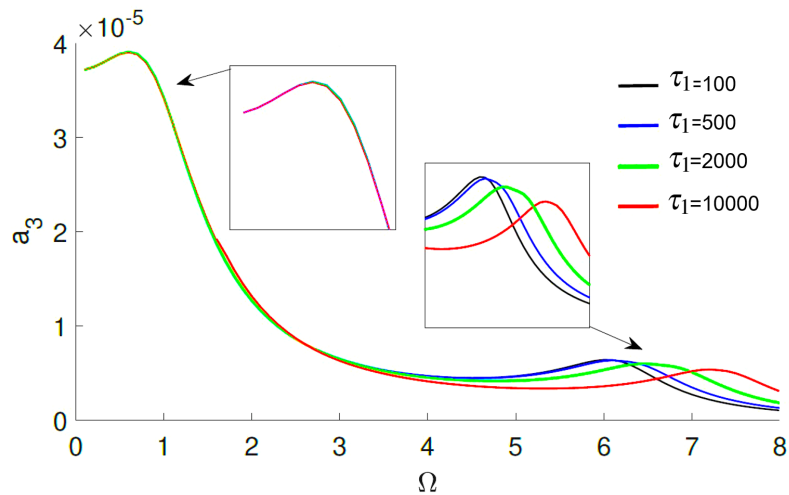


Figure 6.6. Resonance curves of the stapes (a_3) in dimensionless Model 3 for various relaxation times (τ_1).

The changes in resonance peak ($\Omega_2 = 6$) and the stapes amplitude (a_3) are non-linear type, as illustrated in Figure 6.7. When $t_1 > 3000s$, the relaxation effect no longer influences stapes motion; however, for shorter relaxation times, this effect becomes significant and cannot be ignored. Therefore, incorporating the relaxation phenomenon is essential for accurately modeling the middle ear. Notably, the relaxation time does not affect the first resonance. In nonlinear systems, behavior is not always deterministic, meaning the system's response might not be directly proportional to the input parameters, resulting in observable nonlinear phenomena.

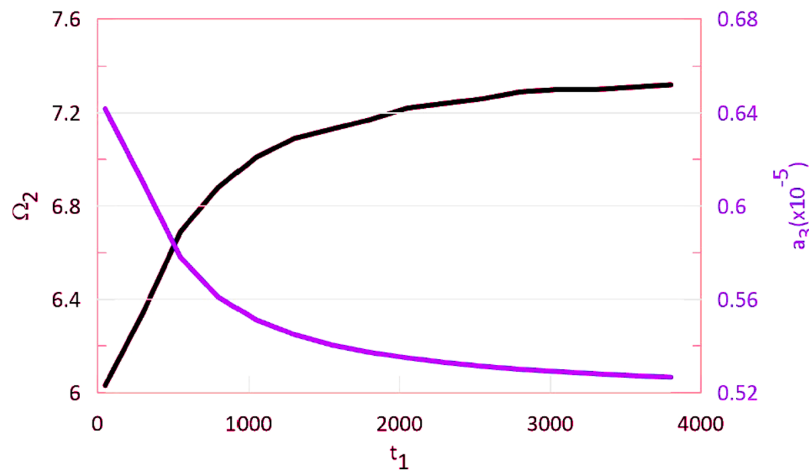


Figure 6.7. Dimensionless second resonance frequency Ω_2 , (black curve) and dimensionless amplitude a_3 , (blue curve) as functions of relaxation time t_1 . The dimensionless frequency Ω_2 is related to ω_2 .

6.5 Discussion

The presented lumped mass model includes certain simplifications, such as assuming a unidirectional horizontal motion of the ossicles, which does not affect the primary vibration modes. The first and second vibration modes of the stapes typically analyzed in the hearing frequency range are longitudinal as reported in key papers from this field [42, 20, 21]. Middle ear dynamics are rarely studied in terms of the impact of joints and ligaments. Most studies assume that human tissue exhibits viscoelastic properties of the K-V type. Occasionally, only FEM models are used to analyze the ossicles' response under various conditions, but rarely account for the relaxation effects typical of human tissue, which should be considered. Our

study is the first to compare the viscoelastic properties of ligaments and joints in the middle ear and their impact on stapes behavior. The three models analyzed here produce the same first resonance as experimental data, but the shift in the second resonance due to the relaxation effect in Model 3 is a new and noteworthy phenomenon. Interestingly, the relaxation time only in Model 3 (where it was analyzed) significantly affects the second resonance. As the relaxation time increases, the resonance frequency rises, and the stapes amplitude decreases. Relaxation time is around 0.01–0.05 seconds, so in real-world dynamic conditions, the shift in the second resonance could be observable in middle ear systems. The relaxation model presented here could be used in the future to model new biomaterials and implants. The Maxwell and Kelvin-Voigt models without relaxation are suitable for analyzing rapidly changing phenomena with short exposure times, where relaxation effects can be neglected. However, it is advisable to use a model with relaxation when exposure time is longer, such as when studying ossicular chain movements under changes in static pressure. Future research should investigate middle ear nonlinearity in both Maxwell and K-V models with relaxation.

The dynamics of the middle ear is sensitive to a variety of conditions and parameters, making accurate modeling particularly important. Finally, the key conclusions drawn from the analysis are as follows:

- Two main resonances of the middle ear are observed in experiments conducted on human temporal bones.
- Experimental tests validate the outputs of the mathematical models demonstrate that the models provide results consistent with the experimental data for the tested preliminary parameters.
- The middle ear models with Maxwell and K-V viscoelasticity yield very similar results, especially when the relaxation time is short and the excitation amplitude is small.
- A longer relaxation time causes a shift in the second resonance in the human middle ear.

- The disturbed harmonic response effect occurs at slightly different external excitation values in the Maxwell viscoelasticity model compared to the Kelvin-Voigt model.
- The analyzed middle ear model with Maxwell-type viscoelasticity is sensitive to the stiffness ratio of additional spring in the system (α), which influences the resonance amplitude.

This study provides important insights into middle ear modeling and can be applied in practice for the development of both passive and active middle ear implants made from new biomaterials. In particular, the active implants should be carefully controlled to avoid disturbances in harmonic responses. The proposed middle ear models can be used to predict middle ear behavior and for control purposes.

IME Analysis

This chapter presents the results of the model simulating an ear with an implant. The model were developed in the same way as mentioned in previous Chapter with the same settings. The analysis focuses on the induced vibrations of the stapes due to the implant's influence. To validate the findings, comparisons were made with both the relevant ASTM F2504 standards and existing literature. Additionally, the previously introduced H_{TV} function was utilized to further assess the system's response. A key observation from this study is the presence of bistability in the stapes vibrations, indicating that the system can settle into two distinct stable states under certain conditions. To better understand this behavior, basins of attraction are constructed within the chapter. These basins illustrate the dependency of the system's response on initial conditions, revealing the parameter ranges in which each stable solution occurs.

Table 7.1. *Parameters of the middle ear used in numerical simulations, taken from experimental validation.*

Stiffness N/m	Damping Ns/m	Other
$k_{TM} = 300$	$c_{TM} = 0.06$	$m_M = 2.5e - 5kg$
$k_{AML} = 800$	$c_{AML} = 0.275$	$m_I = 2.8e - 5kg$
$k_{IMJ} = 100000$	$c_{IMJ} = 0.359$	$m_I = 1.78e - 6kg$
$k_{PIL} = 400$	$c_{PIL} = 0.065$	$M_m = 1.5e - 5kg$
$k_{ISJ} = 1350$	$c_{ISJ} = 0.0079$	$M_m = 1.5e - 5kg$
$k_{AL} = 623$	$c_{AL} = 0.002$	$P = 4.32 * 10^{-5}N$
$k_{AL3} = 1.3e11$	$c_m = 0.32$	$Q = 3.336e - 5N$
$k_C = 200$	$c_C = 0.0017$	
$k_m = 850$	$c_{clip} = 0.15$	
$k_{clip} = 800$		
$k_{clip2} = -2.2e10$		
$k_{clip3} = 6.4e16$		

Above the Table 7.1 with the parameters used for the simulation is shown. As one can observe additional stiffness and damping parameters are presented as well as the new excitation P which corresponds to the implant excitation.

7.1 Frequency Response Functions

At the very begining all of the data that were analyzed in this chapter was collected in one plot (see Fig. 7.1). The yellow region in Figure 7.1 represents the range of results conforming to the ASTM F2504 standard, while the green curve indicates the mean values from 10 measurements. As outlined in the ASTM F2504 standard [3], a single resonance is observed around 1 kHz, with values ranging from 0.250 to 0.062 mm/s/Pa. Beyond 1 kHz, the curves show a gradual decline up to 6 kHz, as depicted in Figure 7.1. The ASTM F2504 standard results are normalized to a pressure of 1 Pa, equivalent to 94 dB SPL.

Figure 7.1 also illustrates the experimentally measured stapes vibration reported in [78] red curve. The experimental data confirm that the intact middle ear's primary resonance occurs at approximately 1 kHz, followed by a second resonance at around 5.5 kHz, which is not accounted for in the ASTM F2504 standard. Nevertheless, our experimental results lie within the tolerance range specified by the

ASTM F2504 standard (green area) within the recommended frequency range of 250–4000 Hz (yellow area).

Second resonance peak is observed not only in 3 DOF IOC model, similar findings are reported in [63, 6, 2], while studies such as [85, 36] identify a second resonance peak near 4 kHz. The presence of an additional peak at 5.5 kHz is numerically validated in the following subsection. Variations between the IOC results in [78, 85] and this study may stem from differences in the mass of reflective tapes, which alter the ossicles' weight, or anatomical differences in temporal bones noted in the literature. To compare numerical results of this study (5-DOF Model) with experimental results, the system is stimulated with the force P to get similar system's output as pressure around 1 Pa applied in the experimental studies. Both numerical and experimental results reveal two resonance peaks. The primary resonance consistently appears around 1 kHz, while the secondary resonance, depending on the model and study, occurs between 4 and 6 kHz. For the 5-DOF implant model, the secondary resonance peak was significantly attenuated compared to the experimental results of the IOC and the 3-DOF IOC model.

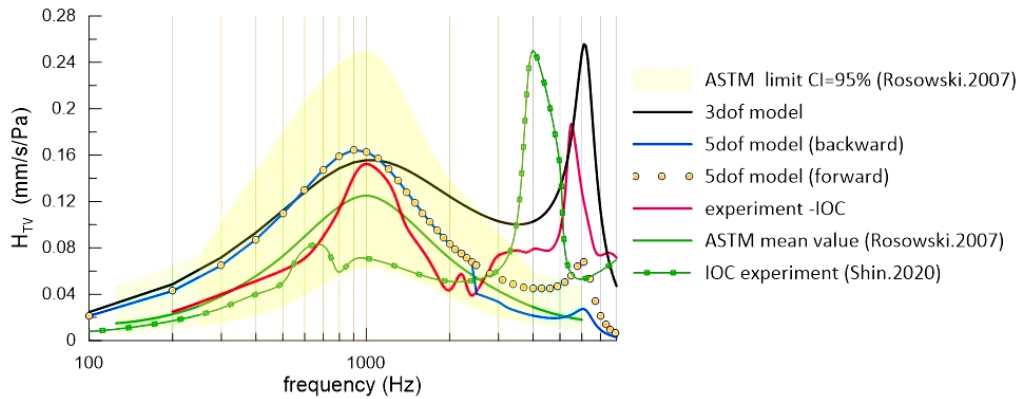


Figure 7.1. Comparison of experimental and numerical results described by the transfer function H_{TV} against ASTM F2504 standards and literature data (results produced by 94 dB SPL).

7.2 Basin of Attraction and Bistability Analysis

The nonlinear model of the IME (5-DOF), introduced here, indicates the possibility of periodic solution bistability near the second resonance, as shown by the

blue curve and yellow dots in Figure 7.1. This phenomenon, not addressed in the ASTM F2504 standard, has been noted in some studies on IMEHDs [13, 81]. The primary resonance peak was observed at 1 kHz, which aligns with the ASTM F2504 limit. However, above 2.5 kHz, the blue curve and yellow dots diverged into two branches (lower and upper), suggesting the existence of two periodic solutions depending on the initial conditions (ICs). This necessitated an analysis of the ICs to determine the basins of attraction (Figures 7.2-7.6). Basin of attraction was created using numerical simulations using Matlab Simulink function called *meshgrid* where two variables was analyzed x - displacement and v -velocity, in analyzing these variables as Initial Conditions for each mass of the system was taken into consideration.

For small initial conditions (blue areas in Figures 7.2-7.6, the resonance curve remained within the ASTM F2504-defined range (blue curve in Figure 7.1). In contrast, larger initial conditions (yellow areas in Figures 7.2-7.6) resulted in H_{TV} values exceeding ASTM F2504 standards above 2.5 kHz (yellow points in Figure 7.1). This underscores the importance of investigating the effects of ICs, including displacement (x) and velocity (v), on system response. Numerical calculations of the basin of attraction at a frequency of $\Omega = 2.5$ kHz were performed.

The IC magnitudes for the first (x_{10}, v_{10}) and second (x_{20}, v_{20}) masses, corresponding to the malleus and incus (Figures 7.2 and 7.3, were on the order of 10^{-4} mm, while the stapes (third mass, x_{30}, v_{30} ; Figure 7.4 exhibited IC magnitudes on the order of 10^{-3} mm. Similarly, the IC magnitudes for the FMT mass (x_{40}, v_{40} ; x_{50}, v_{50} ; Figure 7.5 and 7.6) were approximately 10^{-3} mm. For the implant, velocity-related ICs were also on the order of 10^{-3} mm, whereas displacement-related ICs were approximately 10^{-4} mm.

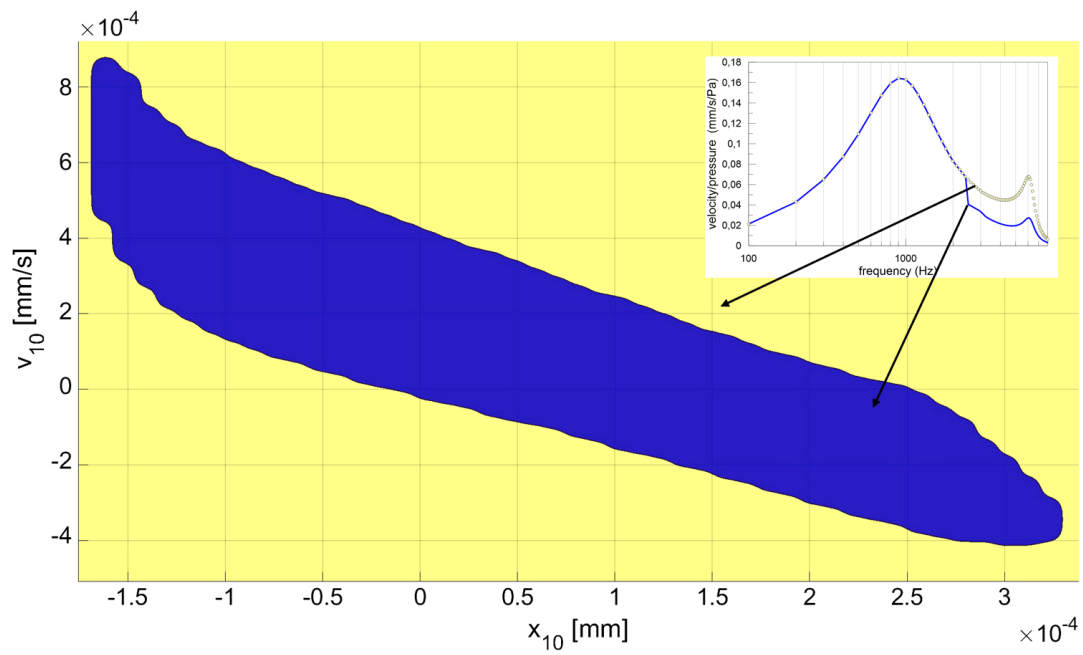


Figure 7.2. The basin of attraction for the stapes motion based on the initial conditions of the malleus (x_{10}, v_{10}) .

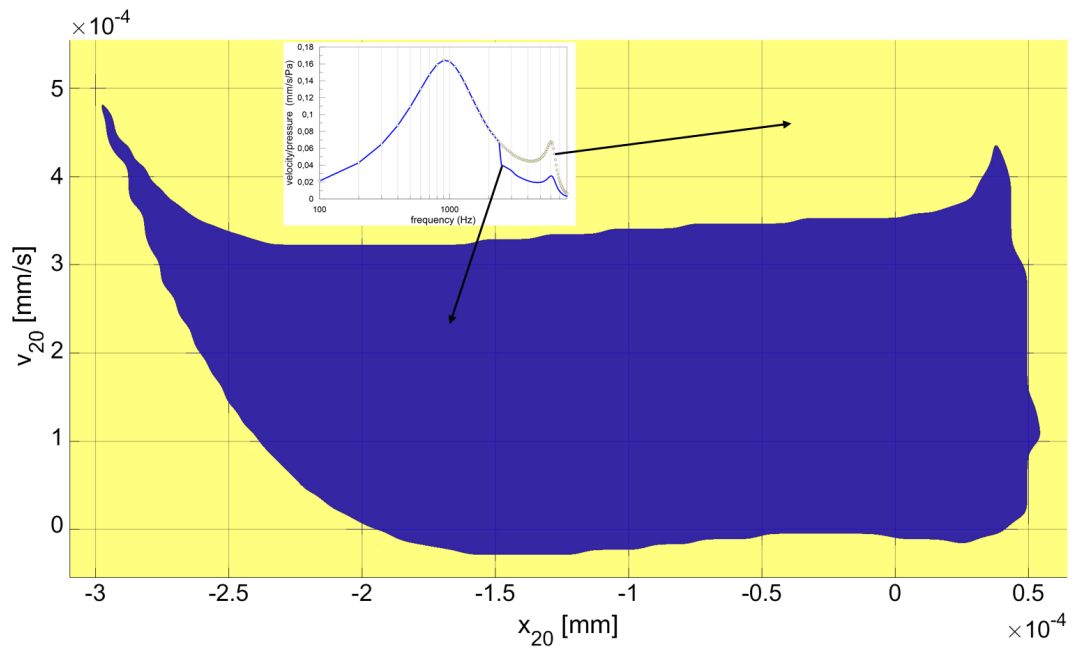


Figure 7.3. The basin of attraction for the stapes motion based on the initial conditions of the incus (x_{20}, v_{20}) .

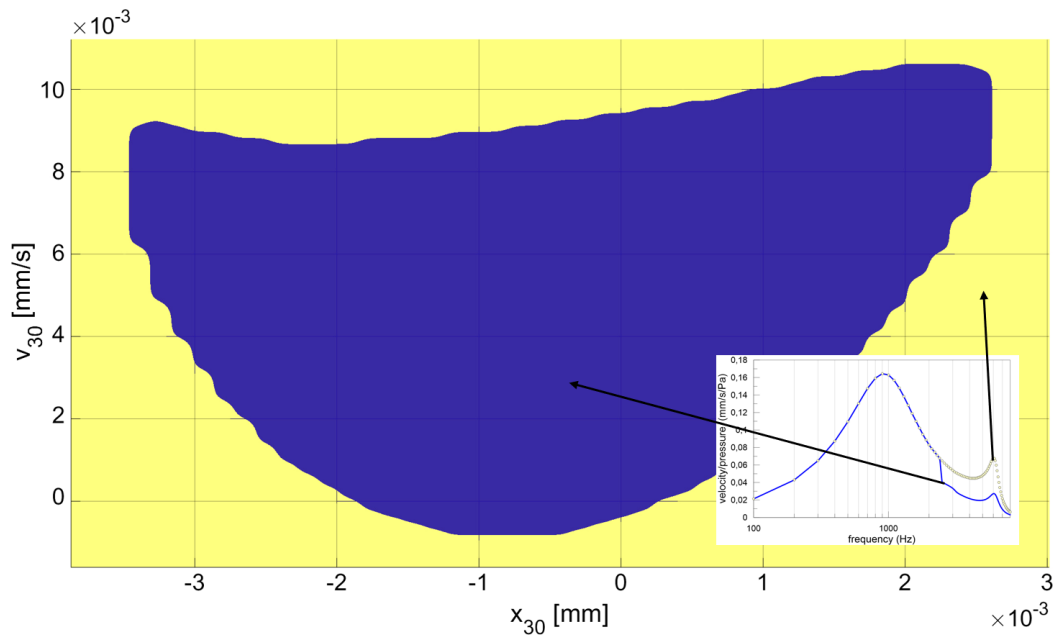


Figure 7.4. The basin of attraction for the stapes motion based on the initial conditions of the stapes (x_{30}, v_{30}) .

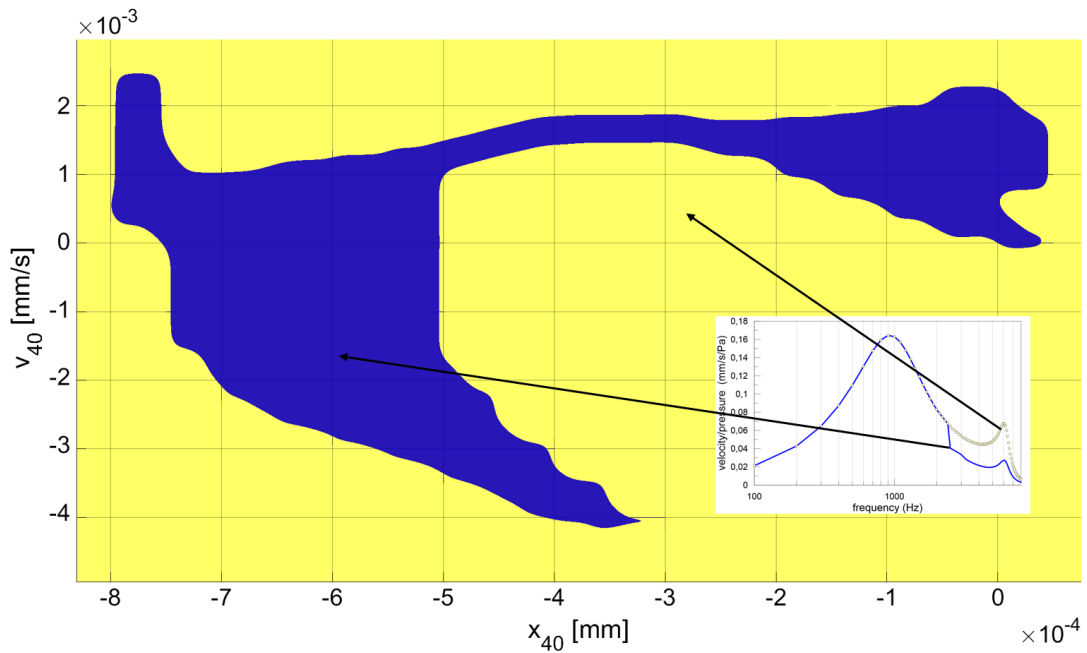


Figure 7.5. The basin of attraction for the stapes motion based on the initial conditions of the case of the FMT (x_{40}, v_{40}) .

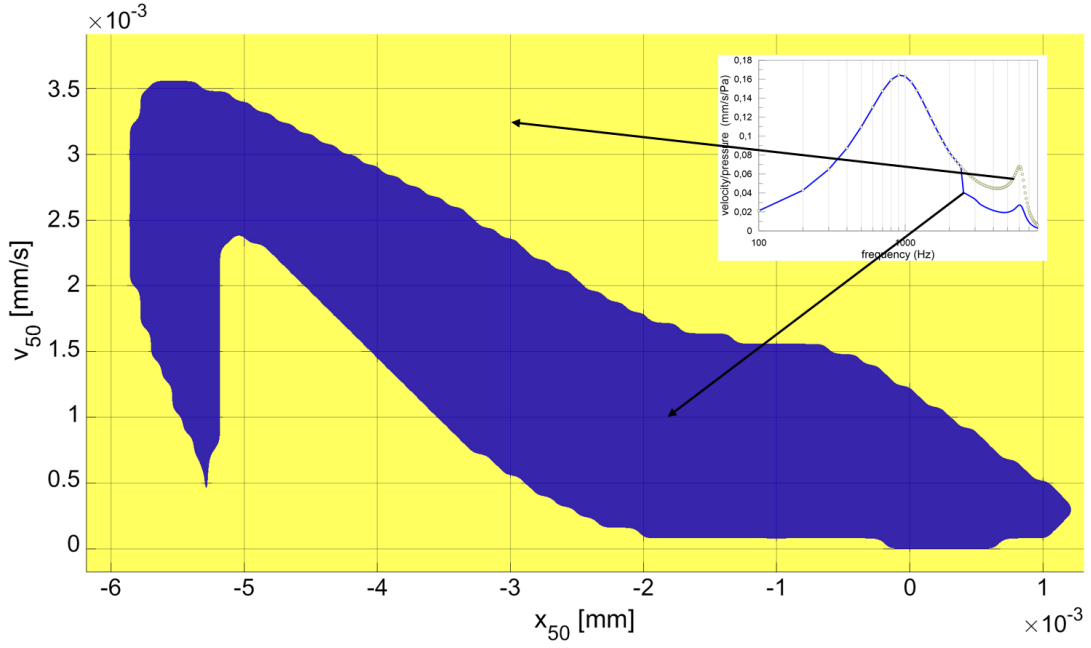


Figure 7.6. The basin of attraction for the stapes motion based on the initial conditions of the mass inside the case (x_{50}, v_{50}) .

The parameters k_{clip} , k_{clip2} , k_{clip3} represent the stiffness of the FMT clip. Of these, k_{clip2} and k_{clip3} are particularly indicative of nonlinear characteristics and are believed to contribute to bifurcation and bistability. Consequently, Figure 7.7 provides a visual representation of the potential (V) of the clip force (F_{clip}), calculated as follows.

$$F_{clip} = k_{clip3}(x_I - x_c)^3 + k_{clip2}(x_I - x_c)^2 + k_{clip}(x_I - x_c) \quad (7.1)$$

$$F_{clip} = \frac{dV}{d(x_I - x_c)} \quad (7.2)$$

$$dV = F_{clip}d(x_I - x_c) \quad (7.3)$$

$$V = \frac{1}{4}k_{clip3}(x_I - x_c)^4 + \frac{1}{3}k_{clip2}(x_I - x_c)^3 + \frac{1}{2}k_{clip}(x_I - x_c)^2 \quad (7.4)$$

The potential curve shown in Figure 7.7 illustrates two distinct potential wells, which are identified as the source of bistability. In this context, bistability refers to the system's ability to reside in two separate stable states (potential wells). This phenomenon is a critical characteristic of many physical systems and holds sig-

nificant implications for the behavior of the implanted human middle ear and its response to external stimuli made by the FMT.

A key finding is the substantial influence of the nonlinear stiffness of the clip, represented by k_{clip2} and k_{clip3} , on the bistability of the middle ear model with five degrees of freedom. This highlights the pivotal role of the clip's stiffness, particularly its nonlinear properties, in shaping the system's dynamic behavior.

These insights could have far-reaching implications for the design, optimization, and maintenance of middle ear implants. They emphasize the necessity of accounting for nonlinear effects in the analysis and design of human middle ear implants to enhance their performance and reliability.

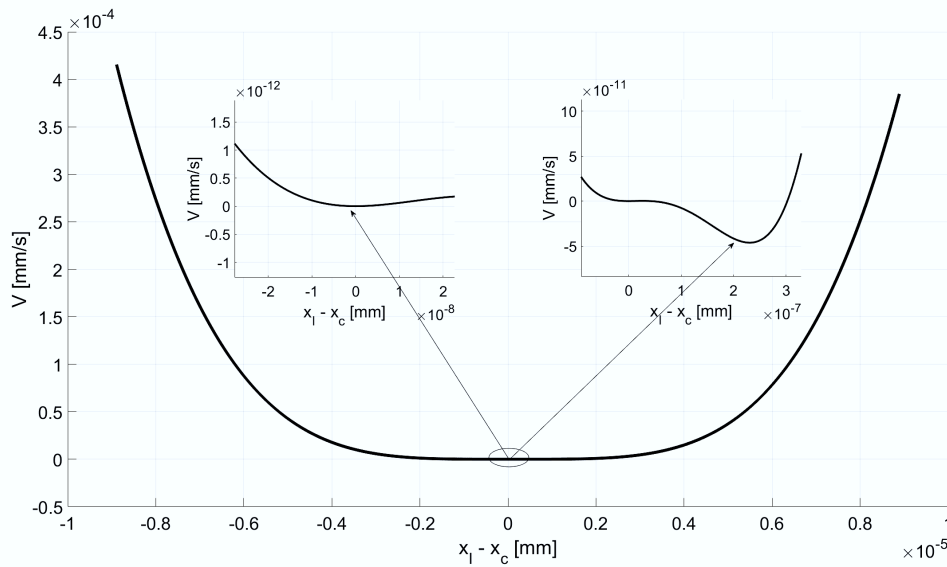


Figure 7.7. Impact of Clip Stiffness on the Bistable Behavior of a 5-DOF Middle Ear Model.

7.3 Discussion

The ASTM F2504 standard specifies the acceptable range of FRF accuracy for IOC vibration induced by sound and measured using the LDV system. Typically, a resonance peak is observed at 1 kHz, corresponding to the primary resonance of the middle ear. Notably, studies by Rusinek et al. [78] and Shin et al. [85] identified resonance peaks not only at 1 kHz but also within the 4 to 6 kHz range — a phenomenon not reported in recent literature. This additional resonance is attributed to bistability in the dynamic system, meaning the system exhibits two

stable equilibrium states depending on its initial conditions. A bistable structure can rest in either of these states and vibrate around the corresponding equilibrium.

Numerical simulations reveal that when initial conditions are near zero, the FRF remains within the ASTM F2504 standard. However, non-zero initial conditions lead to the emergence of a second resonance peak exceeding the limits outlined in Rosowski [74] and the ASTM F2504 standard [3]. This suggests that during experimental measurements, as reported in [78] and [85], non-zero initial conditions may amplify the second resonance peak.

Bistability in the middle ear is infrequently addressed in literature. One notable discussion appears in [77], but the mechanisms driving bistability differ from those in this study. In [77], bistability is likely induced by the piezoelectric element of the FMT. The authors provide a graph illustrating the relationship between bistability, voltage, and frequency, alongside basins of attraction that define initial condition ranges for specific system responses.

While the ASTM F2504 standard sets a test validity range from 0.25 kHz to 4 kHz, it does not exclude phenomena occurring beyond this range, such as the second resonance peak. Identifying bistability under specific initial conditions holds significant implications for the functionality of middle ear implants and auditory processes. The nonlinear stiffness of the implant clip plays a pivotal role in shaping the system's response and stability within a bistable framework. Considering that human hearing and speech frequencies span 0.2 kHz to 8–10 kHz, exceeding the ASTM F2504-defined range, these findings bear practical relevance. This study not only advances theoretical understanding but also provides valuable insights for the design and optimization of middle ear implants.

Double Excited IME Analysis

In this chapter, the model introduced earlier was extended by incorporating a second excitation source, $Q = 3.336e - 5\text{N}$ and additional stiffness parameter $k_{m3} = 1.4e14\text{ N/m}$. The parameters used in the simulations remained consistent with those defined in the previous chapter to ensure comparability. A numerical analysis was conducted, and the results were evaluated against the ASTM F2504 standard to assess compliance and performance.

The study initially focused on cases where both excitations, P and Q , acted in phase. Subsequently, the investigation of the influence of time delays on the system's FRF was conducted. Specifically, the analysis examined the effects of a delayed excitation P —representing the implant's response—where the delay was introduced by the associated electronic system. This step was crucial in understanding how temporal misalignment between excitations impacts the overall dynamic behavior of the model.

8.1 Numerical Analysis

Figure 8.1 illustrates three curves derived from numerical simulations of the model described in the previous section. The results show that for dual excitation using P and Q as specified in Table 7.1, without any time delay, the blue curve remains within the range reported in the literature up to approximately 4 kHz. When excitation is limited to the implant (P), the FRF curve is low but still aligns with the

literature range. Similarly, when excitation is applied solely to the first bone (sound stimulation of the TM via force Q) in the implanted ear, FRF curve reaching their lowest levels between 850 and 1000 Hz. Furthermore, these values fall below the range outlined in [74] (represented by the gray area in Figures 8.1-8.4). All of the curves exceeding mentioned range at 6kHz - second resonance peak.

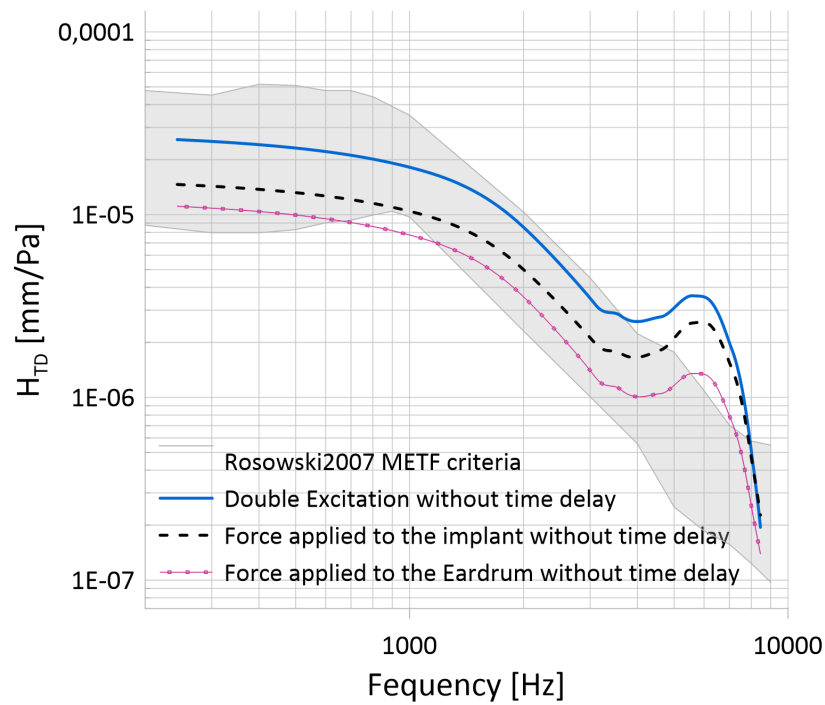


Figure 8.1. Middle Ear Transfer Functions comparison between numerical results (Single excitations without time delay effect) and Rosowski et al. 2007 [74]

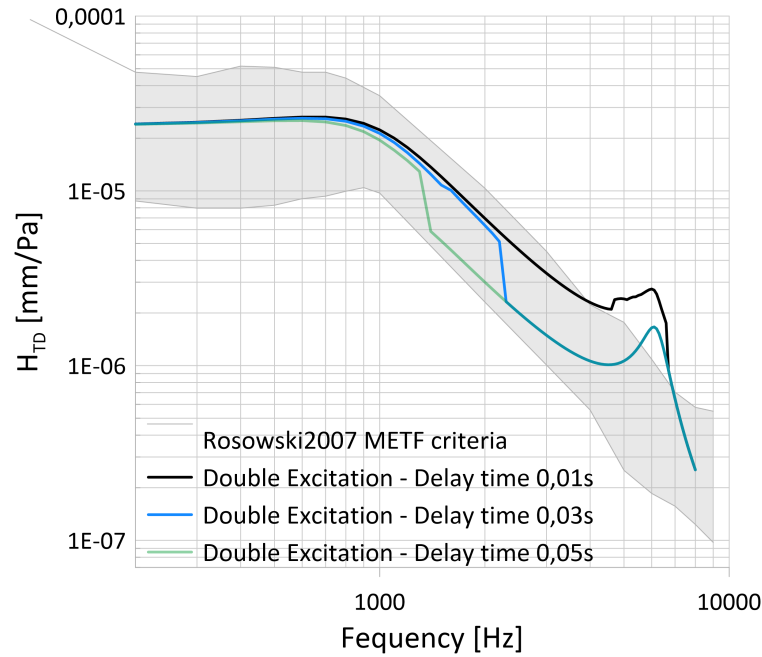


Figure 8.2. Middle Ear Transfer Functions comparison between numerical results (time delay of hundredths of a second) and Rosowski et al. 2007 [74]

In case of double excitation effects of delayed excitation might appear. In this chapter it is considered that the effect of delayed excitation P is applied to the implant, that delays are typical in electronic systems. Figure 8.2 displays FRF curves for the longest delays in this study (τ) of 0.01s, 0.03s, and 0.05s. These curves exhibit a brake in amplitude in FRF at frequencies of 4.5 kHz, 2.2 kHz and 1.5 kHz, and respectively, corresponding to increasing delay durations. Among these, the curves for delays of 0.03s and 0.05s align most closely with the literature and ASTM F2504 criteria.

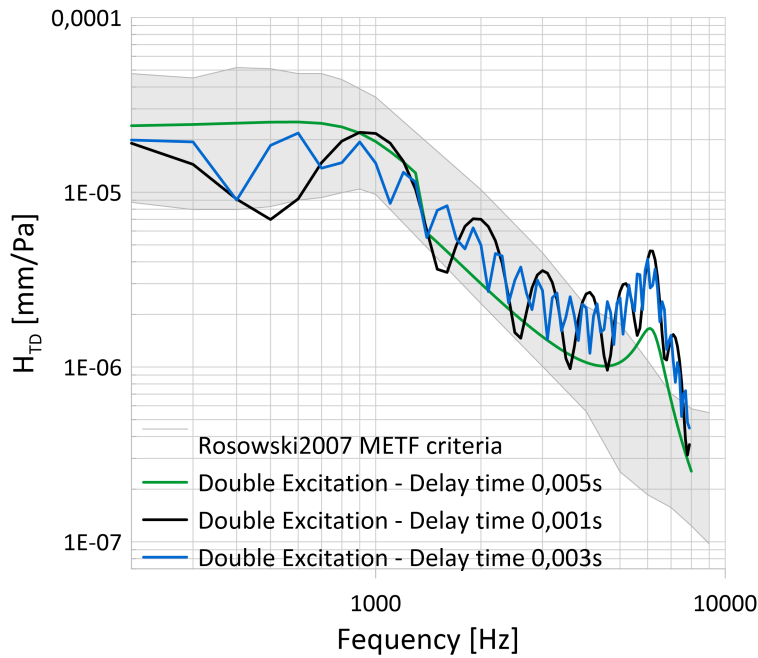


Figure 8.3. Middle Ear Transfer Functions comparison between numerical results (time delay of thousandths of a second) and Rosowski et al. 2007 [74]

In next step the time delay τ of the force P was reduced to thousands of a second. Figure 8.3 presents FRF curves for shorter delay durations of 0.001s, 0.003s, and 0.005s. The green curve, representing a delay of 0.005s, demonstrates the best agreement with the literature, with a single drop near 1.5 kHz. In contrast, the curves for delays of 0.001s and 0.003s (black and blue) show cyclic fluctuations in FRF, with black curve ($\tau = 0.001s$) dropping below the literature range at 0.5 kHz. Notably, the blue curve ($\tau = 0.003s$) fits better in comparison to the black curve inside of the ASTM F2504 range. Interesting part which is cyclic amplitude fluctuations in these two curves will be discussed in the next chapter because this phenomenon need deeper insight.

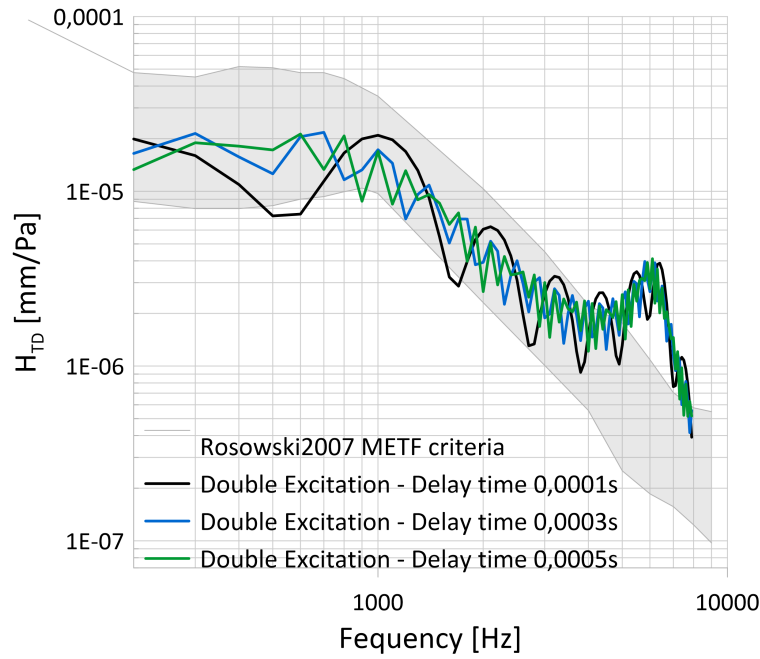


Figure 8.4. Middle Ear Transfer Functions comparison between numerical results (time delay of ten thousandths of a second) and Rosowski et al. 2007 [74]

In the last step of the delay testing the τ of the force P was reduced to tenths of a second. For even shorter delays, as shown in Figure 8.4, the curve for a delay of 0.0003s fits the ASTM F2504 FRF range best, and it is not going below the literature range. The curve for a 0.0001s delay (black) falls below the range at approximately 500-600 Hz, 1.6 kHz. Meanwhile, the green curve for a delay of 0.0005s dips below the range in at 900 Hz.

8.1.1 Delay Effect Analysis

In order to explain phenomenon offluctuations shown in previous section analysis of the signal from numerical simulation was performed. The impact of double excitation and time delay on stapes vibration amplitude (a_3) at an excitation frequency of 1 kHz is depicted in Figure 8.5. This figure demonstrates how the stapes vibration amplitude (a_3) varies with the time delay (τ). The amplitude exhibits cyclic fluctuations, increasing and decreasing periodically as a function of τ . The period of those amplitude changes, is $T = \frac{1}{1000}s$, which is exactly equal to the signal excitation frequency and this is indicated within the figure. Additionally, three

specific points (A, B, C) are highlighted, and their significance is detailed in Figure 8.6-8.8.

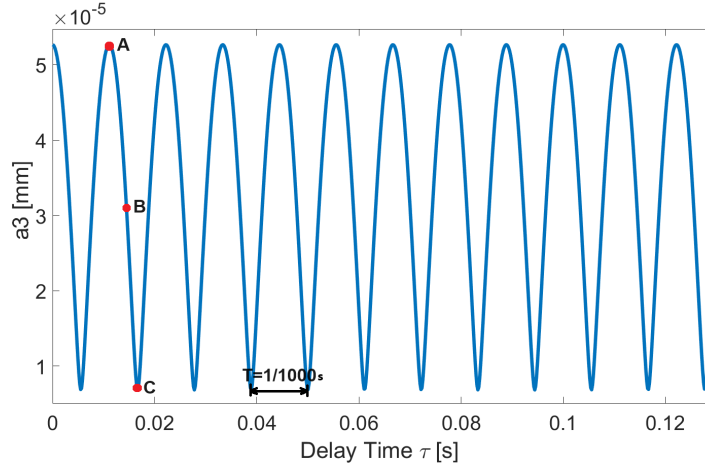


Figure 8.5. *Stapes vibration amplitude based on the delay time (τ) of the excitation force at 1 kHz.*

Figures 8.6-8.8 illustrate three periodic forces: the sound-induced excitation force on the malleus (q), the implant's excitation force from the FMT (p), and their combined effect ($p + q$). These forces are dimensionless in the simulation, defined as $q = \frac{Q}{m_M * x_{st} * \omega_0^2}$ and $p = \frac{P}{m_M * x_{st} * \omega_0^2}$, where $x_{st} = 1[mm]$ and $\omega_0 = \sqrt{\frac{k_{aml}}{m_M}}$, to accelerate calculations.

Figure 8.6, corresponding to point A (when $\tau_1 = 0.011s$ on the amplitude plot in Figure 8.5, shows that both forces are in phase because the delay (τ) equals the excitation period (or a multiple of the period). This synchronization results in the largest combined force ($p + q$).

If time delay $\tau = 0.014s$ and corresponds to point B in the Figure 8.7 the amplitude plot from Figure 8.5, located between the maximum and minimum values. Here, the delay introduces a phase shift between the forces, reducing the resultant force compared to Figure 8.6.

For example when $\tau = 0.017s$ (lowest point on the amplitude graph (Figure 8.5) the forces are shown In Figure 8.8. At this point, the forces are in opposite phases, leading to the smallest combined force ($p + q$).

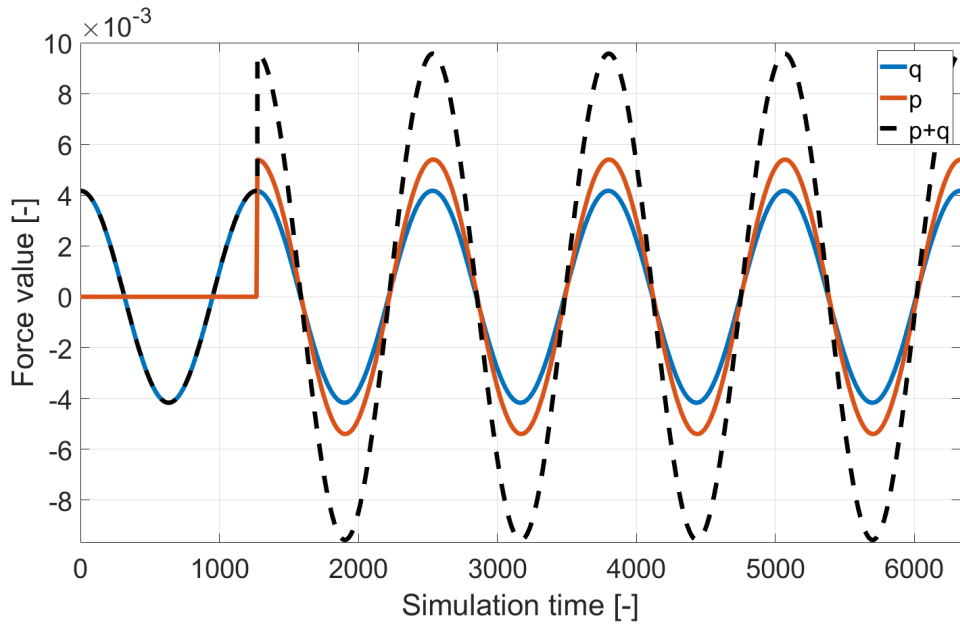


Figure 8.6. Graph of the sinusoidal excitation forces acting on the malleus: from sound stimulation (q), from the implant (p), and their combined effect ($p+q$) corresponding to point A marked in Figure 8.5.

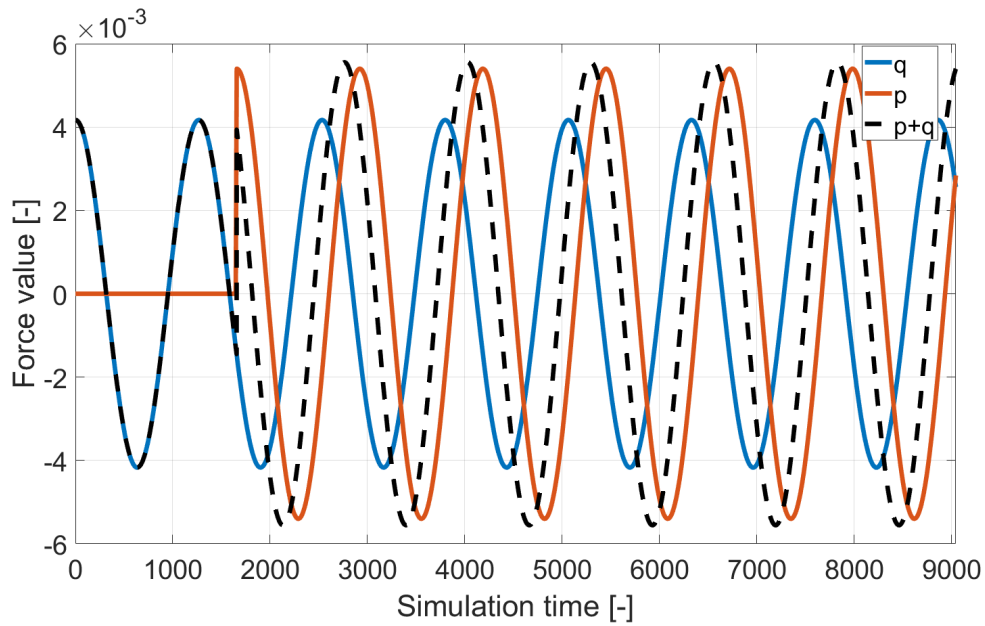


Figure 8.7. Graph of the sinusoidal excitation forces acting on the malleus: from sound stimulation (q), from the implant (p), and their combined effect ($p+q$) corresponding to point B marked in Figure 8.5.

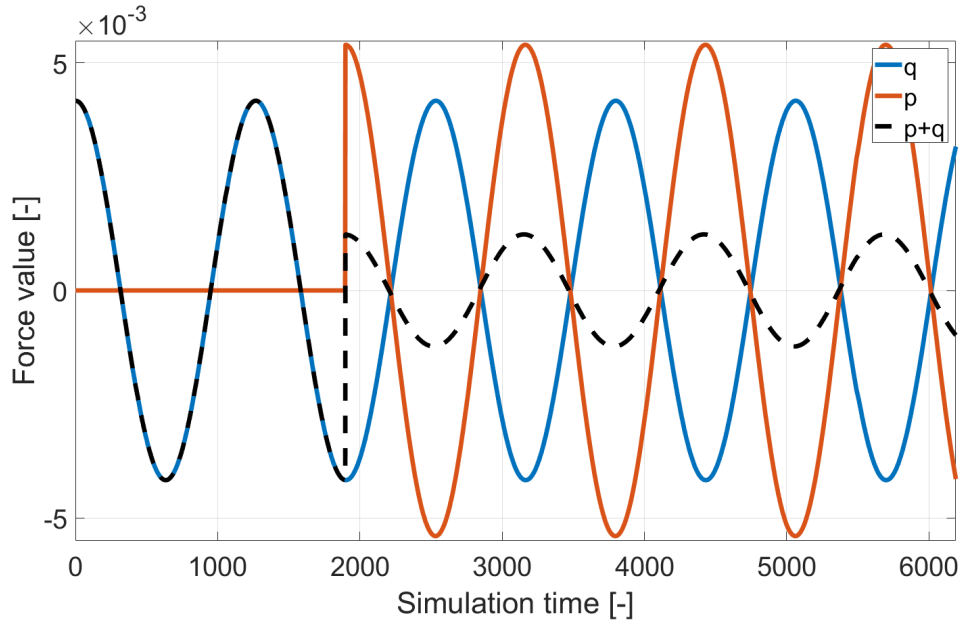


Figure 8.8. Graph of the sinusoidal excitation forces acting on the malleus: from sound stimulation (q), from the implant (p), and their combined effect ($p+q$) corresponding to point C marked in Figure 8.5.

Presented effect of time delay influences the total excitation ($p+q$) and then significantly affect the amplitude graph in Figure 8.5. They also influence the oscillatory behavior seen in Figures 8.6-8.8, where the FRF reflects the phase-dependent summation of forces due to constant time delays and changing excitation frequencies. When the time delay matches the excitation period, stapes vibration is amplified, while a delay of half the excitation period leads to a significant reduction in vibration amplitude.

8.2 Analytical Solution

In this section analytical analysis of the 5 DOF model of IME with double excitation using Harmonic Balance Method (HBM) is shown. The results obtained from analytical solutions are compared to the results from numerical analysis. In table 8.1 the dimensionless parameters are shown. Based on the parameters from the previous section, where double excitation was also analyzed numerically, the dimensionless parameters were calculated to simplify the analysis.

Table 8.1. Dimensionless parameters used for analytical solution.

Stiffness	Damping	Other
$k_{11} = 1251.375$	$c_{11} = 4.9073$	$m_1 = 1$
$k_{12} = -1250$	$c_{12} = -2.5385$	$m_2 = 1.12$
$k_{21} = -1250$	$c_{21} = -2.5385$	$m_3 = 0.0712$
$k_{22} = 1277.2$	$c_{22} = -4.0439$	$m_4 = 0.2$
$k_{23} = -1.6875$	$c_{23} = -0.0559$	$m_5 = 0.2$
$k_{24} = -25$	$c_{24} = -1.0607$	$\beta_{45} = 235$
$k_{32} = -1.6875$	$c_{32} = -0.0559$	$\gamma_3 = 16250$
$k_{33} = 2.7163$	$c_{33} = 0.0682$	$\gamma_{24} = 2812500$
$k_{42} = -25$	$c_{42} = -1.0607$	$\gamma_{45} = 17500$
$k_{44} = 26.0625$	$c_{44} = 3.3234$	$Q = 4.17 \times 10^{-5}$
$k_{45} = -1.0625$	$c_{45} = -2.2627$	$P = 5.40 \times 10^{-5}$
$k_{54} = -1.0625$	$c_{54} = -2.2627$	
$k_{55} = 1.0625$	$c_{55} = 2.2627$	

The equations 8.1 describe a model with two excitations and 5-DOF:

$$\begin{aligned}
& m_1 \ddot{x}_1(t) + k_{11}x_1(t) + k_{12}x_2(t) + c_{11}\dot{x}_1(t) + c_{12}\dot{x}_2(t) - Q \cos(\omega t) = 0 \\
& m_2 \ddot{x}_2(t) + k_{21}x_1(t) + k_{22}x_2(t) + k_{23}x_3(t) + k_{24}x_4(t) + c_{21}\dot{x}_1(t) + c_{22}\dot{x}_2(t) \\
& \quad + c_{23}\dot{x}_3(t) + c_{24}\dot{x}_4(t) + \gamma_{24}(x_2(t) - x_4(t))^3 = 0 \\
& m_3 \ddot{x}_3(t) + k_{32}x_2(t) + k_{33}x_3(t) + c_{32}\dot{x}_2(t) + c_{33}\dot{x}_3(t) + \gamma_3(x_3(t))^3 = 0 \quad (8.1) \\
& m_4 \ddot{x}_4(t) + k_{42}x_2(t) + k_{44}x_4(t) + k_{45}x_5(t) + c_{42}\dot{x}_2(t) + c_{44}\dot{x}_4(t) + c_{45}\dot{x}_5(t) \\
& \quad - \gamma_{24}(x_2(t) - x_4(t))^3 - \beta_{45}(x_4(t) - x_5(t))^2 + \gamma_{45}(x_4(t) - x_5(t))^3 = 0 \\
& m_5 \ddot{x}_5(t) + k_{54}x_4(t) + k_{55}x_5(t) + c_{54}\dot{x}_4(t) + c_{55}\dot{x}_5(t) \\
& \quad \beta_{45}(x_4(t) - x_5(t))^2 - \gamma_{45}(x_4(t) - x_5(t))^3 - P \cos(\omega t) = 0
\end{aligned}$$

The solutions were assumed to be sine and cosine functions and are presented below in the equation 8.2:

$$\begin{aligned}
x_1(t) &= A_{11}(t) \sin(\omega t) + A_{12}(t) \cos(\omega t) \\
\dot{x}_1(t) &= \omega A_{11}(t) \cos(\omega t) - \omega A_{12}(t) \sin(\omega t)
\end{aligned}$$

$$\begin{aligned}
& +\dot{A}_{11}(t) \sin(\omega t) + \dot{A}_{12}(t) \cos(\omega t) \\
\ddot{x}_1(t) = & -\omega^2 A_{11}(t) \sin(\omega t) - \omega^2 A_{12}(t) \cos(\omega t) + 2\dot{A}_{11}(t) \omega \cos(\omega t) \\
& - 2\dot{A}_{12}(t) \omega \sin(\omega t) + \ddot{A}_{11}(t) \sin(\omega t) + \ddot{A}_{12}(t) \cos(\omega t) \\
x_2(t) = & A_{21}(t) \sin(\omega t) + A_{22}(t) \cos(\omega t) \\
\dot{x}_2(t) = & \omega A_{21}(t) \cos(\omega t) - \omega A_{22}(t) \sin(\omega t) \\
& + \dot{A}_{21}(t) \sin(\omega t) + \dot{A}_{22}(t) \cos(\omega t) \\
\ddot{x}_2(t) = & -\omega^2 A_{21}(t) \sin(\omega t) - \omega^2 A_{22}(t) \cos(\omega t) + 2\dot{A}_{21}(t) \omega \cos(\omega t) \\
& - 2\dot{A}_{22}(t) \omega \sin(\omega t) + \ddot{A}_{21}(t) \sin(\omega t) + \ddot{A}_{22}(t) \cos(\omega t) \\
x_3(t) = & A_{31}(t) \sin(\omega t) + A_{32}(t) \cos(\omega t) \\
\dot{x}_3(t) = & \omega A_{31}(t) \cos(\omega t) - \omega A_{32}(t) \sin(\omega t) \quad (8.2) \\
& + \dot{A}_{31}(t) \sin(\omega t) + \dot{A}_{32}(t) \cos(\omega t) \\
\ddot{x}_3(t) = & -\omega^2 A_{31}(t) \sin(\omega t) - \omega^2 A_{32}(t) \cos(\omega t) + 2\dot{A}_{31}(t) \omega \cos(\omega t) \\
& - 2\dot{A}_{32}(t) \omega \sin(\omega t) + \ddot{A}_{31}(t) \sin(\omega t) + \ddot{A}_{32}(t) \cos(\omega t) \\
x_4(t) = & A_{41}(t) \sin(\omega t) + A_{42}(t) \cos(\omega t) \\
\dot{x}_4(t) = & \omega A_{41}(t) \cos(\omega t) - \omega A_{42}(t) \sin(\omega t) \\
& + \dot{A}_{41}(t) \sin(\omega t) + \dot{A}_{42}(t) \cos(\omega t) \\
\ddot{x}_4(t) = & -\omega^2 A_{41}(t) \sin(\omega t) - \omega^2 A_{42}(t) \cos(\omega t) + 2\dot{A}_{41}(t) \omega \cos(\omega t) \\
& - 2\dot{A}_{42}(t) \omega \sin(\omega t) + \ddot{A}_{41}(t) \sin(\omega t) + \ddot{A}_{42}(t) \cos(\omega t) \\
x_5(t) = & A_{51}(t) \sin(\omega t) + A_{52}(t) \cos(\omega t) \\
\dot{x}_5(t) = & \omega A_{51}(t) \cos(\omega t) - \omega A_{52}(t) \sin(\omega t) \\
& + \dot{A}_{51}(t) \sin(\omega t) + \dot{A}_{52}(t) \cos(\omega t) \\
\ddot{x}_5(t) = & -\omega^2 A_{51}(t) \sin(\omega t) - \omega^2 A_{52}(t) \cos(\omega t) + 2\dot{A}_{51}(t) \omega \cos(\omega t) \\
& - 2\dot{A}_{52}(t) \omega \sin(\omega t) + \ddot{A}_{51}(t) \sin(\omega t) + \ddot{A}_{52}(t) \cos(\omega t)
\end{aligned}$$

Equations 8.1 after substituting 8.2 are presented below in equations 8.3 to 8.7:

$$\begin{aligned}
& \cos(\omega t)(-Q + c_{11} \omega A_{11}(t) + (k_{11} - m_1 \omega^2) A_{12}(t) + c_{12} \omega A_{21}(t) \\
& + k_{12} A_{22}(t) + 2m_1 \omega \dot{A}_{11}(t) + c_{11} \dot{A}_{12}(t) + c_{12} \dot{A}_{22}(t) + m_1 \ddot{A}_{12}(t)) \quad (8.3) \\
& + ((k_{11} - m_1 \omega^2) A_{11}(t) - c_{11} \omega A_{12}(t) + k_{12} A_{21}(t) - c_{12} \omega A_{22}(t) + c_{11} \dot{A}_{11}(t) \\
& - 2m_1 \omega \dot{A}_{12}(t) + c_{12} \dot{A}_{21}(t) + m_1 \ddot{A}_{11}(t)) \sin(\omega t) = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4}\gamma_{24}A_{22}(t)^3 \cos(3\omega t) + \frac{3}{4}\gamma_{24}A_{41}(t)^2 A_{42}(t) \cos(3\omega t) \\
& - \frac{1}{4}\gamma_{24}A_{42}(t)^3 \cos(3\omega t) + \cos(\omega t)(c_{21}\omega A_{11}(t) + k_{21}A_{12}(t) + \frac{3}{4}\gamma_{24}A_{22}(t)^3 \\
& + c_{23}\omega A_{31}(t) + k_{23}A_{32}(t) + c_{24}\omega A_{41}(t) + k_{24}A_{42}(t) - \frac{9}{4}\gamma_{24}A_{22}(t)^2 A_{42}(t) \\
& - \frac{3}{4}\gamma_{24}A_{41}(t)^2 A_{42}(t) - \frac{3}{4}\gamma_{24}A_{42}(t)^3 + A_{21}(t)^2(\frac{3}{4}\gamma_{24}A_{22}(t) - \frac{3}{4}\gamma_{24}A_{42}(t)) \\
& + A_{21}(t)(c_{22}\omega - \frac{3}{2}\gamma_{24}A_{22}(t)A_{41}(t) + \frac{3}{2}\gamma_{24}A_{41}(t)A_{42}(t)) \\
& + A_{22}(t)(\frac{1}{4}(4k_{22} - 4m_2\omega^2) + \frac{3}{4}\gamma_{24}A_{41}(t)^2 + \frac{9}{4}\gamma_{24}A_{42}(t)^2) + \frac{1}{4}(4c_{21}\dot{A}_{12}(t) \\
& + 8m_2\omega\dot{A}_{21}(t) + 4c_{22}\dot{A}_{22}(t) + 4c_{23}\dot{A}_{32}(t) + 4c_{24}\dot{A}_{42}(t) + 4m_2\ddot{A}_{22}(t))) \\
& + (k_{21}A_{11}(t) - c_{21}\omega A_{12}(t) + \frac{3}{4}\gamma_{24}A_{21}(t)^3 + k_{23}A_{31}(t) - c_{23}\omega A_{32}(t) \\
& - \frac{9}{4}\gamma_{24}A_{21}(t)^2 A_{41}(t) - \frac{3}{4}\gamma_{24}A_{22}(t)^2 A_{41}(t) - \frac{3}{4}\gamma_{24}A_{41}(t)^3 - c_{24}\omega A_{42}(t) \\
& + A_{22}(t)(-c_{22}\omega + \frac{3}{2}\gamma_{24}A_{41}(t)A_{42}(t)) + A_{41}(t)(k_{24} - \frac{3}{4}\gamma_{24}A_{42}(t)^2) \quad (8.4) \\
& + A_{21}(t)(\frac{1}{4}(4k_{22} - 4m_2\omega^2) + \frac{3}{4}\gamma_{24}A_{22}(t)^2 + \frac{9}{4}\gamma_{24}A_{41}(t)^2 \\
& - \frac{3}{2}\gamma_{24}A_{22}(t)A_{42}(t) + \frac{3}{4}\gamma_{24}A_{42}(t)^2) + \frac{1}{4}(4c_{21}\dot{A}_{11}(t) + 4c_{22}\dot{A}_{21}(t) \\
& - 8m_2\omega\dot{A}_{22}(t) + 4c_{23}\dot{A}_{31}(t) + 4c_{24}\dot{A}_{41}(t) + 4m_2\ddot{A}_{21}(t))) \sin(\omega t) \\
& - \frac{1}{4}\gamma_{24}A_{21}(t)^3 \sin(3\omega t) + \frac{1}{4}\gamma_{24}A_{41}(t)^3 \sin(3\omega t) \\
& - \frac{3}{4}\gamma_{24}A_{41}(t)A_{42}(t)^2 \sin(3\omega t) + A_{22}(t)^2(-(\frac{3}{4}\gamma_{24}A_{42}(t) \cos(3\omega t) \\
& - \frac{3}{4}\gamma_{24}A_{41}(t) \sin(3\omega t)) + A_{21}(t)^2(-(\frac{3}{4}\gamma_{24}A_{22}(t) \cos(3\omega t) \\
& + \frac{3}{4}\gamma_{24}A_{42}(t) \cos(3\omega t) + \frac{3}{4}\gamma_{24}A_{41}(t) \sin(3\omega t)) \\
& + A_{22}(t)(-(\frac{3}{4}\gamma_{24}A_{41}(t)^2 \cos(3\omega t) + \frac{3}{4}\gamma_{24}A_{42}(t)^2 \cos(3\omega t) \\
& + \frac{3}{2}\gamma_{24}A_{41}(t)A_{42}(t) \sin(3\omega t)) + A_{21}(t)(-(\frac{3}{2}\gamma_{24}A_{41}(t)A_{42}(t) \cos(3\omega t) \\
& + \frac{3}{4}\gamma_{24}A_{22}(t)^2 \sin(3\omega t) - \frac{3}{4}\gamma_{24}A_{41}(t)^2 \sin(3\omega t) \\
& + \frac{3}{4}\gamma_{24}A_{42}(t)^2 \sin(3\omega t) + A_{22}(t)(\frac{3}{2}\gamma_{24}A_{41}(t) \cos(3\omega t) \\
& - \frac{3}{2}\gamma_{24}A_{42}(t) \sin(3\omega t))) = 0
\end{aligned}$$

$$\begin{aligned}
& -\left(\frac{3}{4}\right)\gamma_3 A_{31}(t)^2 A_{32}(t) \cos(3\omega t) + \frac{1}{4}\gamma_3 A_{32}(t)^3 \cos(3\omega t) + \cos(\omega t)(c_{32}\omega A_{21}(t) \\
& + k_{32}A_{22}(t) + c_{33}\omega A_{31}(t) + \frac{1}{4}(4k_{33} - 4m_3\omega^2)A_{32}(t) + \frac{3}{4}\gamma_3 A_{31}(t)^2 A_{32}(t) \\
& + \frac{3}{4}\gamma_3 A_{32}(t)^3 + \frac{1}{4}(4c_{32}\dot{A}_{22}(t) + 8m_3\omega\dot{A}_{31}(t) \\
& + 4c_{33}\dot{A}_{32}(t) + 4m_3\ddot{A}_{32}(t))) + (k_{32}A_{21}(t) - c_{32}\omega A_{22}(t) + \frac{3}{4}\gamma_3 A_{31}(t)^3 \\
& - c_{33}\omega A_{32}(t) + A_{31}(t)(\frac{1}{4}(4k_{33} - 4m_3\omega^2) + \frac{3}{4}\gamma_3 A_{32}(t)^2) \\
& + \frac{1}{4}(4c_{32}\dot{A}_{21}(t) + 4c_{33}\dot{A}_{31}(t) - 8m_3\omega\dot{A}_{32}(t) + 4m_3\ddot{A}_{31}(t))) \sin(\omega t) \\
& - \frac{1}{4}\gamma_3 A_{31}(t)^3 \sin(3\omega t) + \frac{3}{4}\gamma_3 A_{31}(t)A_{32}(t)^2 \sin(3\omega t) = 0
\end{aligned} \tag{8.5}$$

$$\begin{aligned}
& \frac{1}{4}A_{52}(t)^2(-2\beta_{45} - 2\beta_{45}\cos(2\omega t)) - \frac{1}{4}\gamma_{24}A_{22}(t)^3 \cos(3\omega t) \\
& - \frac{1}{4}\gamma_{45}A_{52}(t)^3 \cos(3\omega t) + \frac{1}{4}A_{42}(t)^3(\gamma_{24}\cos(3\omega t) + \gamma_{45}\cos(3\omega t)) \\
& + A_{51}(t)^2(\frac{1}{4}(-2\beta_{45} + 2\beta_{45}\cos(2\omega t)) + \frac{3}{4}\gamma_{45}A_{52}(t)\cos(3\omega t)) \\
& + \cos(\omega t)(-\frac{3}{4}\gamma_{24}A_{22}(t)^3 + \frac{9}{4}\gamma_{24}A_{22}(t)^2 A_{42}(t) \\
& + \frac{1}{4}(3\gamma_{24} + 3\gamma_{45})A_{42}(t)^3 + A_{21}(t)^2(-\frac{3}{4}\gamma_{24}A_{22}(t) \\
& + \frac{3}{4}\gamma_{24}A_{42}(t)) + A_{21}(t)(c_{42}\omega + \frac{3}{2}\gamma_{24}A_{22}(t)A_{41}(t) \\
& - \frac{3}{2}\gamma_{24}A_{41}(t)A_{42}(t)) + A_{22}(t)(k_{42} - \frac{3}{4}\gamma_{24}A_{41}(t)^2 \\
& - \frac{9}{4}\gamma_{24}A_{42}(t)^2) + c_{45}\omega A_{51}(t) + k_{45}A_{52}(t) - \frac{9}{4}\gamma_{45}A_{42}(t)^2 A_{52}(t) \\
& - \frac{3}{4}\gamma_{45}A_{51}(t)^2 A_{52}(t) - \frac{3}{4}\gamma_{45}A_{52}(t)^3 + A_{41}(t)^2(\frac{1}{4}(3\gamma_{24} + 3\gamma_{45})A_{42}(t) \\
& - \frac{3}{4}\gamma_{45}A_{52}(t)) + A_{41}(t)(c_{44}\omega - \frac{3}{2}\gamma_{45}A_{42}(t)A_{51}(t) \\
& + \frac{3}{2}\gamma_{45}A_{51}(t)A_{52}(t)) + A_{42}(t)(\frac{1}{4}(4k_{44} - 4m_4\omega^2) \\
& + \frac{3}{4}\gamma_{45}A_{51}(t)^2 + \frac{9}{4}\gamma_{45}A_{52}(t)^2) + \frac{1}{4}(4c_{42}\dot{A}_{22}(t) + 8m_4\omega\dot{A}_{41}(t) \\
& + 4c_{44}\dot{A}_{42}(t) + 4c_{45}\dot{A}_{52}(t) + 4m_4\ddot{A}_{42}(t))) + (-\frac{3}{4}\gamma_{24}A_{21}(t)^3
\end{aligned}$$

$$\begin{aligned}
& +\frac{9}{4}\gamma_{24}A_{21}(t)^2A_{41}(t) + \frac{3}{4}\gamma_{24}A_{22}(t)^2A_{41}(t) \\
& +\frac{1}{4}(3\gamma_{24} + 3\gamma_{45})A_{41}(t)^3 + A_{22}(t)(-c_{42}\omega - \frac{3}{2}\gamma_{24}A_{41}(t)A_{42}(t)) \\
& +A_{21}(t)(k_{42} - \frac{3}{4}\gamma_{24}A_{22}(t)^2 - \frac{9}{4}\gamma_{24}A_{41}(t)^2 + \frac{3}{2}\gamma_{24}A_{22}(t)A_{42}(t) \\
& -\frac{3}{4}\gamma_{24}A_{42}(t)^2) - \frac{9}{4}\gamma_{45}A_{41}(t)^2A_{51}(t) - \frac{3}{4}\gamma_{45}A_{42}(t)^2A_{51}(t) - \frac{3}{4}\gamma_{45}A_{51}(t)^3 \\
& -c_{45}\omega A_{52}(t) + A_{42}(t)(-c_{44}\omega + \frac{3}{2}\gamma_{45}A_{51}(t)A_{52}(t)) \\
& +A_{51}(t)(k_{45} - \frac{3}{4}\gamma_{45}A_{52}(t)^2) + A_{41}(t)(\frac{1}{4}(4k_{44} - 4m_4\omega^2) \\
& +\frac{1}{4}(3\gamma_{24} + 3\gamma_{45})A_{42}(t)^2 + \frac{9}{4}\gamma_{45}A_{51}(t)^2 - \frac{3}{2}\gamma_{45}A_{42}(t)A_{52}(t) \\
& +\frac{3}{4}\gamma_{45}A_{52}(t)^2) + \frac{1}{4}(4c_{42}\dot{A}_{21}(t) + 4c_{44}\dot{A}_{41}(t) - 8m_4\omega\dot{A}_{42}(t) \\
& +4c_{45}\dot{A}_{51}(t) + 4m_4\ddot{A}_{41}(t))) \sin(\omega t) + \frac{1}{4}\gamma_{24}A_{21}(t)^3 \sin(3\omega t) \quad (8.6) \\
& +\frac{1}{4}\gamma_{45}A_{51}(t)^3 \sin(3\omega t) + \frac{1}{4}A_{41}(t)^3(-\gamma_{24} \sin(3\omega t) \\
& -\gamma_{45} \sin(3\omega t)) + A_{21}(t)^2(\frac{3}{4}\gamma_{24}A_{22}(t) \cos(3\omega t) \\
& -\frac{3}{4}\gamma_{24}A_{42}(t) \cos(3\omega t) - \frac{3}{4}\gamma_{24}A_{41}(t) \sin(3\omega t)) \\
& +A_{22}(t)^2(\frac{3}{4}\gamma_{24}A_{42}(t) \cos(3\omega t) + \frac{3}{4}\gamma_{24}A_{41}(t) \sin(3\omega t)) \\
& +A_{22}(t)(\frac{3}{4}\gamma_{24}A_{41}(t)^2 \cos(3\omega t) - \frac{3}{4}\gamma_{24}A_{42}(t)^2 \cos(3\omega t) \\
& -\frac{3}{2}\gamma_{24}A_{41}(t)A_{42}(t) \sin(3\omega t)) + A_{42}(t)^2(\frac{1}{4}(-2\beta_{45} - 2\beta_{45} \cos(2\omega t)) \\
& -\frac{3}{4}\gamma_{45}A_{52}(t) \cos(3\omega t) - \frac{3}{4}\gamma_{45}A_{51}(t) \sin(3\omega t)) \\
& +A_{41}(t)^2(\frac{1}{4}(-2\beta_{45} + 2\beta_{45} \cos(2\omega t)) + \frac{3}{4}\gamma_{45}A_{52}(t) \cos(3\omega t) \\
& +\frac{1}{4}A_{42}(t)(-3\gamma_{24} \cos(3\omega t) - 3\gamma_{45} \cos(3\omega t)) + \frac{3}{4}\gamma_{45}A_{51}(t) \sin(3\omega t)) \\
& +A_{51}(t)(-\beta_{45}A_{52}(t) \sin(2\omega t) - \frac{3}{4}\gamma_{45}A_{52}(t)^2 \sin(3\omega t)) \\
& +A_{21}(t)(\frac{3}{2}\gamma_{24}A_{41}(t)A_{42}(t) \cos(3\omega t) - \frac{3}{4}\gamma_{24}A_{22}(t)^2 \sin(3\omega t) \\
& +\frac{3}{4}\gamma_{24}A_{41}(t)^2 \sin(3\omega t) - \frac{3}{4}\gamma_{24}A_{42}(t)^2 \sin(3\omega t) \\
& +A_{22}(t)(-\frac{3}{2}\gamma_{24}A_{41}(t) \cos(3\omega t) + \frac{3}{2}\gamma_{24}A_{42}(t) \sin(3\omega t)))
\end{aligned}$$

$$\begin{aligned} & +A_{41}(t)(A_{51}(t)(\frac{1}{4}(4\beta_{45} - 4\beta_{45}\cos(2\omega t)) - \frac{3}{2}\gamma_{45}A_{52}(t)\cos(3\omega t)) \\ & \quad + \beta_{45}A_{52}(t)\sin(2\omega t) - \frac{3}{4}\gamma_{45}A_{51}(t)^2\sin(3\omega t) \\ & \quad + \frac{3}{4}\gamma_{45}A_{52}(t)^2\sin(3\omega t) + \frac{1}{4}A_{42}(t)^2(3\gamma_{24}\sin(3\omega t) \\ & \quad + 3\gamma_{45}\sin(3\omega t)) + A_{42}(t)(\frac{3}{2}\gamma_{45}A_{51}(t)\cos(3\omega t) - \beta_{45}\sin(2\omega t) \\ & \quad - \frac{3}{2}\gamma_{45}A_{52}(t)\sin(3\omega t))) + A_{42}(t)(\frac{1}{4}A_{52}(t)(4\beta_{45} + 4\beta_{45}\cos(2\omega t)) \\ & \quad - \frac{3}{4}\gamma_{45}A_{51}(t)^2\cos(3\omega t) + \frac{3}{4}\gamma_{45}A_{52}(t)^2\cos(3\omega t) \\ & \quad + A_{51}(t)(\beta_{45}\sin(2\omega t) + \frac{3}{2}\gamma_{45}A_{52}(t)\sin(3\omega t))) = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{4}A_{52}(t)^2(2\beta_{45} + 2\beta_{45}\cos(2\omega t)) - \frac{1}{4}\gamma_{45}A_{42}(t)^3\cos(3\omega t) \\ & + \frac{1}{4}\gamma_{45}A_{52}(t)^3\cos(3\omega t) + A_{51}(t)^2(\frac{1}{4}(2\beta_{45} - 2\beta_{45}\cos(2\omega t)) \\ & \quad - \frac{3}{4}\gamma_{45}A_{52}(t)\cos(3\omega t)) + \frac{1}{4}\gamma_{45}A_{41}(t)^3\sin(3\omega t) \\ & - \frac{1}{4}\gamma_{45}A_{51}(t)^3\sin(3\omega t) + A_{41}(t)^2(\frac{1}{4}(2\beta_{45} - 2\beta_{45}\cos(2\omega t)) \\ & \quad + \frac{3}{4}\gamma_{45}A_{42}(t)\cos(3\omega t) - \frac{3}{4}\gamma_{45}A_{52}(t)\cos(3\omega t) \\ & - \frac{3}{4}\gamma_{45}A_{51}(t)\sin(3\omega t)) + A_{42}(t)^2(\frac{1}{4}(2\beta_{45} + 2\beta_{45}\cos(2\omega t)) \\ & \quad + \frac{3}{4}\gamma_{45}A_{52}(t)\cos(3\omega t) + \frac{3}{4}\gamma_{45}A_{51}(t)\sin(3\omega t)) \\ & + A_{51}(t)(\beta_{45}A_{52}(t)\sin(2\omega t) + \frac{3}{4}\gamma_{45}A_{52}(t)^2\sin(3\omega t)) \\ & + A_{42}(t)(\frac{1}{4}A_{52}(t)(-4\beta_{45} - 4\beta_{45}\cos(2\omega t)) + \frac{3}{4}\gamma_{45}A_{51}(t)^2\cos(3\omega t) \\ & \quad - \frac{3}{4}\gamma_{45}A_{52}(t)^2\cos(3\omega t) + A_{51}(t)(-\beta_{45}\sin(2\omega t) \\ & - \frac{3}{2}\gamma_{45}A_{52}(t)\sin(3\omega t))) + A_{41}(t)(A_{51}(t)(\frac{1}{4}(-4\beta_{45} + 4\beta_{45}\cos(2\omega t)) \\ & \quad + \frac{3}{2}\gamma_{45}A_{52}(t)\cos(3\omega t)) - \beta_{45}A_{52}(t)\sin(2\omega t) \\ & \quad - \frac{3}{4}\gamma_{45}A_{42}(t)^2\sin(3\omega t) + \frac{3}{4}\gamma_{45}A_{51}(t)^2\sin(3\omega t) \\ & \quad - \frac{3}{4}\gamma_{45}A_{52}(t)^2\sin(3\omega t) + A_{42}(t)(-\frac{3}{2}\gamma_{45}A_{51}(t)\cos(3\omega t) \end{aligned}$$

$$\begin{aligned}
& +\beta_{45} \sin(2\omega t) + \frac{3}{2}\gamma_{45}A_{52}(t) \sin(3\omega t)) \quad (8.7) \\
& +\cos(\omega t)\left(-\left(\frac{3}{4}\right)\gamma_{45}A_{42}(t)^3 + c_{55}\omega A_{51}(t) + \frac{1}{4}(4k_{55} - 4m_5\omega^2)A_{52}(t) \right. \\
& \quad +\frac{9}{4}\gamma_{45}A_{42}(t)^2A_{52}(t) + \frac{3}{4}\gamma_{45}A_{51}(t)^2A_{52}(t) \\
& \quad +\frac{3}{4}\gamma_{45}A_{52}(t)^3 + A_{41}(t)^2\left(-\left(\frac{3}{4}\right)\gamma_{45}A_{42}(t) \right. \\
& \quad +\frac{3}{4}\gamma_{45}A_{52}(t)) + A_{41}(t)\left(\frac{3}{2}\gamma_{45}A_{42}(t)A_{51}(t) - \frac{3}{2}\gamma_{45}A_{51}(t)A_{52}(t) \right. \\
& \quad +\omega c_{54}) + \frac{1}{4}(-4P + 8m_5\omega\dot{A}_{51}(t) + 4c_{55}\dot{A}_{52}(t) + 4m_5\ddot{A}_{52}(t) \\
& \quad +4\dot{A}_{42}(t)c_{54}) + A_{42}(t)\left(-\left(\frac{3}{4}\right)\gamma_{45}A_{51}(t)^2 - \frac{9}{4}\gamma_{45}A_{52}(t)^2 + k_{54} \right. \\
& \quad +\sin(\omega t)\left(-\left(\frac{3}{4}\right)\gamma_{45}A_{41}(t)^3 + \frac{9}{4}\gamma_{45}A_{41}(t)^2A_{51}(t) \right. \\
& \quad +\frac{3}{4}\gamma_{45}A_{42}(t)^2A_{51}(t) + \frac{3}{4}\gamma_{45}A_{51}(t)^3 - c_{55}\omega A_{52}(t) \\
& \quad +A_{51}(t)\left(\frac{1}{4}(4k_{55} - 4m_5\omega^2) + \frac{3}{4}\gamma_{45}A_{52}(t)^2) + A_{42}(t)\left(-\left(\frac{3}{2}\right)\gamma_{45}A_{51}(t)A_{52}(t) \right. \\
& \quad -\omega c_{54}) + \frac{1}{4}(4c_{55}\dot{A}_{51}(t) - 8m_5\omega\dot{A}_{52}(t) + 4m_5\ddot{A}_{51}(t) + 4\dot{A}_{41}(t)c_{54}) \\
& \quad +A_{41}(t)\left(-\left(\frac{3}{4}\right)\gamma_{45}A_{42}(t)^2 - \frac{9}{4}\gamma_{45}A_{51}(t)^2 + \frac{3}{2}\gamma_{45}A_{42}(t)A_{52}(t) \right. \\
& \quad \left. \left. -\frac{3}{4}\gamma_{45}A_{52}(t)^2 + k_{54}\right) = 0
\end{aligned}$$

Then the equations were rearranged and the coefficients for sine 8.8-8.12 and cosine 8.13-8.17 were extracted and compared to zero (the order of equations is consistent with the equations above):

$$\begin{aligned}
& k_{11}A_{11}(t) - m_1\omega^2A_{11}(t) - c_{11}\omega A_{12}(t) + k_{12}A_{21}(t) - c_{12}\omega A_{22}(t) \quad (8.8) \\
& +c_{11}\dot{A}_{11}(t) - 2m_1\omega\dot{A}_{12}(t) + c_{12}\dot{A}_{21}(t) + m_1\ddot{A}_{11}(t) = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4}(4k_{21}A_{11}(t) - 4c_{21}\omega A_{12}(t) + 4k_{21}A_{21}(t) - 4m_2\omega^2A_{21}(t) \\
& +3\gamma_{24}A_{21}(t)^3 - 4c_{22}\omega A_{22}(t) + 3\gamma_{24}A_{21}(t)A_{22}(t)^2 + 4k_{23}A_{31}(t) \\
& -4c_{23}\omega A_{32}(t) + 4k_{24}A_{41}(t) - 9\gamma_{24}A_{21}(t)^2A_{41}(t) - 3\gamma_{24}A_{22}(t)^2A_{41}(t)
\end{aligned}$$

$$\begin{aligned}
& +9\gamma_{24}A_{21}(t)A_{41}(t)^2 - 3\gamma_{24}A_{41}(t)^3 - 4c_{24}\omega A_{42}(t) - 6\gamma_{24}A_{21}(t)A_{22}(t)A_{42}(t) \quad (8.9) \\
& +6\gamma_{24}A_{22}(t)A_{41}(t)A_{42}(t) + 3\gamma_{24}A_{21}(t)A_{42}(t)^2 - 3\gamma_{24}A_{41}(t)A_{42}(t)^2 \\
& +4c_{21}\dot{A}_{11}(t) + 4c_{22}\dot{A}_{21}(t) - 8m_2\omega\dot{A}_{22}(t) + 4c_{23}\dot{A}_{31}(t) \\
& +4c_{24}\dot{A}_{41}(t) + 4m_2\ddot{A}_{21}(t)) = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4}(4k_{32}A_{21}(t) - 4c_{32}\omega A_{22}(t) + 4k_{33}A_{31}(t) - 4m_3\omega^2 A_{31}(t) \\
& +3\gamma_3 A_{31}(t)^3 - 4c_{33}\omega A_{32}(t) + 3\gamma_3 A_{31}(t)A_{32}(t)^2 + 4c_{32}\dot{A}_{21}(t) \quad (8.10) \\
& +4c_{33}\dot{A}_{31}(t) - 8m_3\omega\dot{A}_{32}(t) + 4m_3\ddot{A}_{31}(t)) = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4}(4k_{42}A_{21}(t) - 3\gamma_{24}A_{21}(t)^3 - 4c_{42}\omega A_{22}(t) - 3\gamma_{24}A_{21}(t)A_{22}(t)^2 \\
& +4k_{44}A_{41}(t) - 4m_4\omega^2 A_{41}(t) + 9\gamma_{24}A_{21}(t)^2 A_{41}(t) + 3\gamma_{24}A_{22}(t)^2 A_{41}(t) \\
& -9\gamma_{24}A_{21}(t)A_{41}(t)^2 + 3\gamma_{24}A_{41}(t)^3 + 3\gamma_{45}A_{41}(t)^3 - 4c_{44}\omega A_{42}(t) \\
& +6\gamma_{24}A_{21}(t)A_{22}(t)A_{42}(t) - 6\gamma_{24}A_{22}(t)A_{41}(t)A_{42}(t) - 3\gamma_{24}A_{21}(t)A_{42}(t)^2 \quad (8.11) \\
& +3\gamma_{24}A_{41}(t)A_{42}(t)^2 + 3\gamma_{45}A_{41}(t)A_{42}(t)^2 + 4k_{45}A_{51}(t) - 9\gamma_{45}A_{41}(t)^2 A_{51}(t) \\
& -3\gamma_{45}A_{42}(t)^2 A_{51}(t) + 9\gamma_{45}A_{41}(t)A_{51}(t)^2 - 3\gamma_{45}A_{51}(t)^3 - 4c_{45}\omega A_{52}(t) \\
& -6\gamma_{45}A_{41}(t)A_{42}(t)A_{52}(t) + 6\gamma_{45}A_{42}(t)A_{51}(t)A_{52}(t) + 3\gamma_{45}A_{41}(t)A_{52}(t)^2 \\
& -3\gamma_{45}A_{51}(t)A_{52}(t)^2 + 4c_{42}\dot{A}_{21}(t) + 4c_{44}\dot{A}_{41}(t) - 8m_4\omega\dot{A}_{42}(t) \\
& +4c_{45}\dot{A}_{51}(t) + 4m_4\ddot{A}_{41}(t)) = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4}(4k_{54}A_{41}(t) - 3\gamma_{45}A_{41}(t)^3 - 4c_{54}\omega A_{42}(t) - 3\gamma_{45}A_{41}(t)A_{42}(t)^2 \\
& +4k_{55}A_{51}(t) - 4m_5\omega^2 A_{51}(t) + 9\gamma_{45}A_{41}(t)^2 A_{51}(t) + 3\gamma_{45}A_{42}(t)^2 A_{51}(t) \\
& -9\gamma_{45}A_{41}(t)A_{51}(t)^2 + 3\gamma_{45}A_{51}(t)^3 - 4c_{55}\omega A_{52}(t) + 6\gamma_{45}A_{41}(t)A_{42}(t)A_{52}(t) \quad (8.12) \\
& -6\gamma_{45}A_{42}(t)A_{51}(t)A_{52}(t) - 3\gamma_{45}A_{41}(t)A_{52}(t)^2 + 3\gamma_{45}A_{51}(t)A_{52}(t)^2 \\
& +4c_{54}\dot{A}_{41}(t) + 4c_{55}\dot{A}_{51}(t) - 8m_5\omega\dot{A}_{52}(t) + 4m_5\ddot{A}_{51}(t)) = 0
\end{aligned}$$

$$\begin{aligned} -Q + c_{11}\omega A_{11}(t) + k_{11}A_{12}(t) - m_1\omega^2 A_{12}(t) + c_{12}\omega A_{21}(t) + k_{12}A_{22}(t) \quad (8.13) \\ + 2m_1\omega\dot{A}_{11}(t) + c_{11}\dot{A}_{12}(t) + c_{12}\dot{A}_{22}(t) + m_1\ddot{A}_{11}(t) = 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{4}(4c_{21}\omega A_{11}(t) + 4k_{21}A_{12}(t) + 4c_{22}\omega A_{21}(t) + 4k_{21}A_{22}(t) \\ - 4m_2\omega^2 A_{22}(t) + 3\gamma_{24}A_{21}(t)^2 A_{22}(t) + 3\gamma_{24}A_{22}(t)^3 \\ + 4c_{23}\omega A_{31}(t) + 4k_{23}A_{32}(t) + 4c_{24}\omega A_{41}(t) - 6\gamma_{24}A_{21}(t)A_{22}(t)A_{41}(t) \\ + 3\gamma_{24}A_{22}(t)A_{41}(t)^2 + 4k_{24}A_{42}(t) - 3\gamma_{24}A_{21}(t)^2 A_{42}(t) \quad (8.14) \\ - 9\gamma_{24}A_{22}(t)^2 A_{42}(t) + 6\gamma_{24}A_{21}(t)A_{41}(t)A_{42}(t) - 3\gamma_{24}A_{41}(t)^2 A_{42}(t) \\ + 9\gamma_{24}A_{22}(t)A_{42}(t)^2 - 3\gamma_{24}A_{42}(t)^3 + 4c_{21}\dot{A}_{12}(t) + 8m_2\omega\dot{A}_{21}(t) \\ + 4c_{22}\dot{A}_{22}(t) + 4c_{23}\dot{A}_{32}(t) + 4c_{24}\dot{A}_{42}(t) + 4m_2\ddot{A}_{22}(t)) = 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{4}(4c_{32}\omega A_{21}(t) + 4k_{32}A_{22}(t) + 4c_{33}\omega A_{31}(t) + 4k_{33}A_{32}(t) \\ - 4m_3\omega^2 A_{32}(t) + 3\gamma_{34}A_{31}(t)^2 A_{32}(t) + 3\gamma_{34}A_{32}(t)^3 + 4c_{32}\dot{A}_{22}(t) \quad (8.15) \\ + 8m_3\omega\dot{A}_{31}(t) + 4c_{33}\dot{A}_{32}(t) + 4m_3\ddot{A}_{32}(t)) = 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{4}(4c_{42}\omega A_{21}(t) + 4k_{42}A_{22}(t) - 3\gamma_{24}A_{21}(t)^2 A_{22}(t) - 3\gamma_{24}A_{22}(t)^3 \\ + 4c_{44}\omega A_{41}(t) + 6\gamma_{24}A_{21}(t)A_{22}(t)A_{41}(t) - 3\gamma_{24}A_{22}(t)A_{41}(t)^2 \\ + 4k_{44}A_{42}(t) - 4m_4\omega^2 A_{42}(t) + 3\gamma_{24}A_{21}(t)^2 A_{42}(t) + 9\gamma_{24}A_{22}(t)^2 A_{42}(t) \\ - 6\gamma_{24}A_{21}(t)A_{41}(t)A_{42}(t) + 3\gamma_{24}A_{41}(t)^2 A_{42}(t) + 3\gamma_{45}A_{41}(t)^2 A_{42}(t) \\ - 9\gamma_{24}A_{22}(t)A_{42}(t)^2 + 3\gamma_{24}A_{42}(t)^3 + 3\gamma_{45}A_{42}(t)^3 + 4c_{45}\omega A_{51}(t) \\ - 6\gamma_{45}A_{41}(t)A_{42}(t)A_{51}(t) + 3\gamma_{45}A_{42}(t)A_{51}(t)^2 + 4k_{45}A_{52}(t) \quad (8.16) \\ - 3\gamma_{45}A_{41}(t)^2 A_{52}(t) - 9\gamma_{45}A_{42}(t)^2 A_{52}(t) + 6\gamma_{45}A_{41}(t)A_{51}(t)A_{52}(t) \\ - 3\gamma_{45}A_{51}(t)^2 A_{52}(t) + 9\gamma_{45}A_{42}(t)A_{52}(t)^2 - 3\gamma_{45}A_{52}(t)^3 + 4c_{42}\dot{A}_{22}(t) \\ + 8m_4\omega\dot{A}_{41}(t) + 4c_{44}\dot{A}_{42}(t) + 4c_{45}\dot{A}_{52}(t) + 4m_4\ddot{A}_{42}(t)) = 0 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{4}(-4P + 4c_{54}\omega A_{41}(t) + 4k_{54}A_{42}(t) - 3\gamma_{45}A_{41}(t)^2A_{42}(t) \\
& - 3\gamma_{45}A_{42}(t)^3 + 4c_{55}\omega A_{51}(t) + 6\gamma_{45}A_{41}(t)A_{42}(t)A_{51}(t) \\
& - 3\gamma_{45}A_{42}(t)A_{51}(t)^2 + 4k_{55}A_{52}(t) - 4m_5\omega^2A_{52}(t) + 3\gamma_{45}A_{41}(t)^2A_{52}(t) \\
& + 9\gamma_{45}A_{42}(t)^2A_{52}(t) - 6\gamma_{45}A_{41}(t)A_{51}(t)A_{52}(t) \quad (8.17) \\
& + 3\gamma_{45}A_{51}(t)^2A_{52}(t) - 9\gamma_{45}A_{42}(t)A_{52}(t)^2 + 3\gamma_{45}A_{52}(t)^3 + 4c_{54}\dot{A}_{42}(t) \\
& + 8m_5\omega\dot{A}_{51}(t) + 4c_{55}\dot{A}_{52}(t) + 4m_5\ddot{A}_{52}(t)) = 0
\end{aligned}$$

Next, the higher harmonics are compared to zero as $\ddot{A}_{ij} = 0$ and $\dot{A}_{ij} = 0$. The final form of the amplitudes are shown in equation 8.18:

$$\begin{aligned}
amp_1 &= \sqrt{A_{11}^2 + A_{12}^2} \\
amp_2 &= \sqrt{A_{21}^2 + A_{22}^2} \\
amp_3 &= \sqrt{A_{31}^2 + A_{32}^2} \\
amp_4 &= \sqrt{A_{41}^2 + A_{42}^2} \\
amp_5 &= \sqrt{A_{51}^2 + A_{52}^2}
\end{aligned} \quad (8.18)$$

Based on the above calculations, graphs were obtained and compared with the graphs obtained by numerical simulation for each of the amplitudes, (malleus (amp_1), incus (amp_2), stapes (amp_3), FMT case (amp_4) and FMT mass (amp_5)).

This section presents frequency–amplitude response curves obtained from both numerical and analytical methods. The numerical results were generated using MATLAB Simulink, while the analytical solutions were derived using the HBM.

Figure 8.9a displays two amplitude curves for the malleus. The close overlap of these curves indicates strong agreement between the numerical and analytical results. A single resonance peak is observed at approximately 700 Hz, beyond which the amplitude decreases.

Figure 8.9b illustrates the amplitude response for the incus. The curves here are nearly identical to those for the malleus (Figure 8.9), which can be attributed to the very rigid connection between these two bones in the anatomical structure—modeled as coupled masses in the simulation.

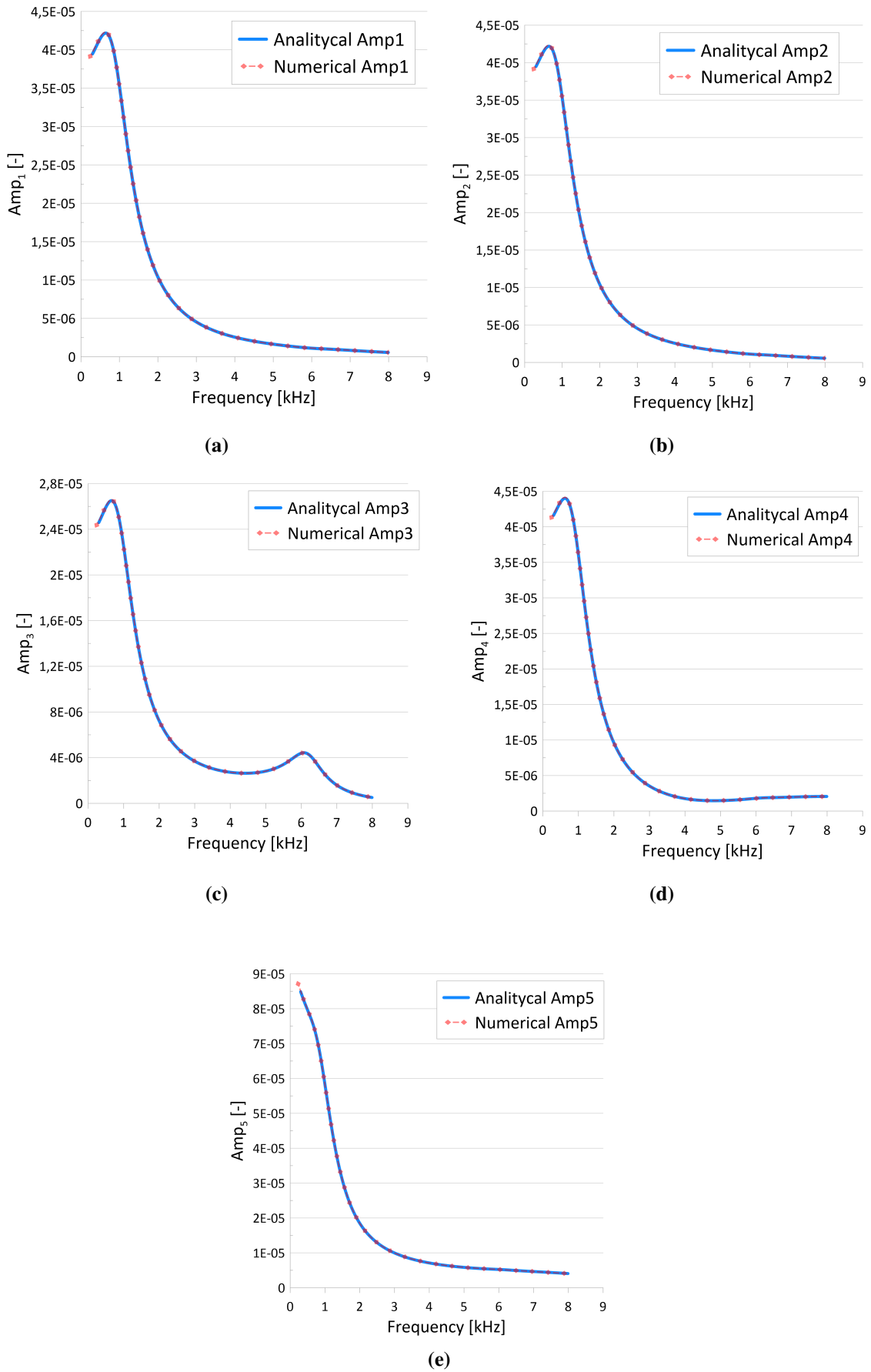


Figure 8.9. Comparison between analytical and numerical results of the amplitudes for: a) malleus, b) incus, c) stapes, d) case of the FMT and e) FMT mass.

In the case of the stapes, shown in Figure 8.9c, two distinct resonance peaks are observed. The first occurs at approximately 700 Hz, similar to the malleus and incus, while the second peak appears around 6000 Hz.

Figure 8.9d shows the response of the FMT case. A single resonance peak is present, and the amplitude declines until roughly 5000 Hz, followed by a slight increase.

For the FMT mass (Figure 8.9e), the first resonance peak is not visible on the amplitude–frequency curve. This omission is likely due to the fact that both numerical and analytical calculations begin at 200 Hz, which corresponds to the lower threshold of human hearing. For all masses, the amplitude-frequency curves have very good agreement between the numerical and analytical results.

8.3 Discussion

Existing literature do not analysis of the double excitation phenomenon in implanted ears stimulated by the FMT, even though experimental findings, such as those by Schraven et al. [81] and Shin et al. [85], indicate fluctuations in the FRF. Studies [81, 85], which examine stapes motion driven by the FMT, reveal irregularities in the FRFs, unlike the smoother patterns observed in numerical and experimental results for unaided ears (without the FMT). This suggests that the observed fluctuations are likely caused by the interaction of double excitation and the time delay introduced by FMT. In contrast to findings reported in studies [76] [78], the model under investigation here does not exhibit nonlinear behaviors; stapes vibrations remain regular throughout. Figures 8.1-8.8 emphasize the importance of accounting for potential time delays in the system when configuring the implant. Double excitation can introduce amplitude fluctuations that fall below the FRF range defined in [74]. Recognizing and mitigating these delays are essential for optimizing implant functionality and ensuring precise sound transmission. Proper consideration and configuration of delay times can significantly reduce disturbances, enhance implant performance, and improve auditory outcomes for users. Notably, other studies using lumped parameter models of the ear with implants, such as [76, 78] do not address the effects of double excitation or the impact of

time delays in one of the excitation forces, leaving a gap in the understanding of this phenomenon.

The analytical solution performed using the HBM successfully captures the dynamic response of the ossicle chain in the human middle ear under double excitation. The comparison between analytical and numerical results, particularly the amplitude–frequency response curves for each of the five degrees of freedom, reveals a strong correlation in both resonance behavior and amplitude trends.

For the malleus and incus, the results demonstrate almost identical amplitude profiles, with a prominent resonance peak at approximately 700 Hz. This outcome validates the assumption of a rigid mechanical coupling between these two ossicles, consistent with their anatomical configuration. The analytical solutions accurately predict the resonance which corresponds with the numerical simulation, suggesting that the model reliably simulates the low-frequency dynamics of the middle ear.

The stapes, represented by the third mass, exhibits a more complex response with two clear resonance peaks—around 700 Hz and 6000 Hz. The FMT case and FMT mass, corresponding to the fourth and fifth masses, respectively, show amplitude responses that further support the model’s robustness. Although the first resonance peak is not visible for the FMT mass—due to the lower frequency limit of 200 Hz—its overall response aligns well with the simulated data. This indicates that the analytical model remains valid even in scenarios where mechanical input is applied externally to the ossicle chain (e.g., via implantable transducers).

Overall, the close agreement between analytical and numerical results across all five components confirms the validity and accuracy of the Harmonic Balance approach for this class of biomechanical systems. The analytical formulation provides an efficient alternative to computationally expensive time-domain simulations and can be used for further optimization and parametric studies of IMEHD.

Final Conclusions

This doctoral dissertation presents a comprehensive investigation into the biomechanics of the human middle ear, focusing on both intact and implanted states. The research involved extensive experimental and numerical analyses to elucidate the behavior of the ossicle chain under various conditions, including normal functionality, prosthetic implantation, and the effects of double excitation with time delay. The primary contributions and findings of this research are summarized below.

1. Intact Middle Ear Analysis

- The middle ear exhibits two primary resonance frequencies: the first resonance appears around 1 kHz, while a second resonance is observed in the range of 4–6 kHz, which is absent in ASTM standard expectations.
- The Maxwell and Kelvin-Voigt viscoelastic models provide a reasonable approximation of the middle ear behavior, capturing the observed experimental dynamics effectively. Both systems give similar answers, which makes it impossible to indicate a better model.
- The nonlinearities of the ossicle chain, particularly in response to high excitation levels, lead to disturbed harmonic responses. This phenomenon manifests as deviations from purely harmonic oscillations and suggests the need for careful consideration of excitation amplitude when assessing middle ear function.

- Stiffness ratios and relaxation times significantly influence the dynamic response of the stapes, particularly affecting the second resonance peak and its amplitude.
- The research provides new insights into the viscoelastic behavior of ligaments and joints in the middle ear, challenging the assumptions made in earlier biomechanical studies.

2. Implanted Middle Ear Analysis

A major component of this dissertation focused on implanted middle ears, where IMEHD alter the dynamics of sound transmission. Key findings include:

- The addition of an implant introduces mass loading effects, which modify the frequency response of the middle ear. In particular, the first resonance peak is significantly dampened, while the second resonance persists, albeit with variations depending on implant design and attachment site.
- Bistability was observed in the implanted middle ear system, particularly near the second resonance. This phenomenon, influenced by the clip stiffness of the implant, results in two stable states of vibration depending on initial conditions.
- The nonlinear stiffness of implant fixation clips plays a crucial role in system dynamics, making careful material selection and mechanical tuning essential for optimal implant performance.
- While the ASTM standard provides a useful benchmark for implant performance, it fails to account for the second resonance and potential bistability effects. Future standards should incorporate these additional dynamical behaviors.

3. Double Excitation and Time Delay Effects

The study also examined the effects of simultaneous excitations—one from natural acoustic stimulation and the other from an active implant—along with the impact of time delays. Key insights include:

- The interaction between two excitation sources leads to periodic fluctuations in the FRF. These fluctuations are influenced by the relative phase

shifts introduced by time delays in the electronic control of active implants.

- When the phase difference between the two excitations approaches zero, the stapes vibration amplitude is maximized due to constructive interference. Conversely, when the excitations are out of phase, destructive interference reduces stapes displacement, potentially impairing hearing perception.
- The presence of a second resonance peak is further affected by double excitation conditions, requiring careful calibration of active implants to prevent undesirable fluctuations in frequency response.
- Shorter time delays (on the order of milliseconds) cause cyclic variations in FRF, with implications for IMEDs. Longer delays result in frequency-dependent amplitude reductions, further complicating implant tuning.
- The findings highlight the need for improved control algorithms in active implants that compensate for phase shifts and optimize auditory outcomes.

4. Practical Implications and Future Research

The outcomes of this dissertation have several implications for middle ear research and medical applications:

- **Hearing Implant Design:** The research findings provide valuable insights for engineers developing middle ear implants, particularly in optimizing attachment mechanisms, controlling nonlinear effects, and mitigating bistability.
- **Diagnostic Tools:** The refined understanding of resonance shifts and transfer function deviations in implanted ears could enhance clinical diagnostics, aiding otologists in selecting and adjusting prosthetic devices for individual patients.
- **ASTM Standards Improvement:** The ASTM standard, which currently overlooks certain nonlinearities and bistability effects, should be revised to include parameters addressing these phenomena

- **Numerical Modeling:** Future research should further refine numerical models by integrating additional viscoelastic effects and improving simulation techniques to better match experimental observations.
- **Extended Clinical Studies:** While this research provides a strong foundation, larger clinical studies are necessary to validate the identified behaviors across diverse patient groups and implant types.

In summary, this dissertation has provided a detailed biomechanical analysis of the human middle ear, shedding light on the interplay between natural and prosthetic dynamics. The findings challenge conventional assumptions, introduce new perspectives on nonlinear behavior, and pave the way for advancements in hearing restoration technologies. By bridging the gap between experimental and numerical approaches, this work lays the foundation for advancements in middle ear implants and diagnostic methodologies, ultimately contributing to improved auditory health outcomes and expanding knowledge within the field of mechanical engineering.

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